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Inflation Decomposition Model: Application to Macedonian inflation

Danica Unevska Andonova*

Abstract

The purpose of the paper is to introduce the framework for decomposing the forecast of headline inflation, obtained by macroeconomic model of NBRM for monetary policy analysis and medium term projections (MAKPAM), into its components: food, energy and core inflation. The model for inflation decomposition is a small structural model, set up in state space framework. Kalman filter procedure is applied to filter the future paths of CPI components, given projected headline inflation obtained by MAKPAM model and exogenous determinants, such as output gap, world commodity prices, and foreign effective inflation. The results of the model's forecasting performance suggest that this model can be a useful analytical tool in the process of inflation forecast, with relatively good fit of equations for food and domestic oil prices. This model serves as satellite model to MAKPAM and enriches the set of tools for forecasting and monetary policy analysis in NBRM. In this paper we highlight its most important equations, results and model performances.

JEL classifications: C53, E31, E37
Keywords: Inflation, forecasting, Macedonia

* Monetary Policy and Research Department, National Bank of the Republic of Macedonia. Corresponding author: UnevskaD@nbrm.mk. The responsibility for this paper lies solely with the individual authors. The views expressed herein do not necessarily represent those of the National Bank of the Republic of Macedonia. I would like to thank Rilind Kabashi who contributed in the development of the model, with his expertise and valuable comments.

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1. Introduction

The price stability is the main objective for most of the central banks, including the National bank of the Republic of Macedonia (NBRM). Development of inflation is crucial for monetary policy, particularly the forecast of future inflation, as the transmission of monetary policy decisions to real economy occurs with time delay. Therefore, the projection of inflation in central banks plays important role. NBRM also pays a lot of attention to inflation forecasting, and there are set of different methods, for near term and medium term inflation forecasting. There are suits of models for near term forecasting, starting from simple time series models to small structural model (Petrovska et al., 2017). Currently, NBRM is using the following models for short-term forecasting of inflation: ARIMA models (aggregated and disaggregated approach), dynamic factor model and a small (three equation) structural model. The medium term inflation forecast is obtained by Macedonian Policy Analysis Model (MAKPAM), which is a consistent framework of the transmission mechanisms in the Macedonian economy (Hledik et al., 2016). Within this model, the inflation process is modeled through New Keynesian, forward looking, Phillips curve. In line with the theory, the equation for consumer price inflation (without administered prices) is defined as a function of expected inflation, lagged inflation and the real marginal cost, which is weighted sum of the output gap (domestic inflation pressures) and imported prices (imported inflation pressures).

However, the policymakers and the general public usually follow inflation developments by Consumer price index (CPI) components, such as food, energy and core inflation, which are published by state statistical offices on monthly basis. However, these CPI components are not directly connected with inflation equation in MAKPAM model. Additionally, the policymakers would be interested which inflation component contributes the most to the total inflation on medium term and according to that whether should react or not. For example, if the domestic prices are driven by exogenous factors such as world oil prices, to which monetary policy has no direct control, than it is more likely the monetary authorities not to react to such shocks, which are usually perceived as temporary shocks. Thus, the forecasters need to explain and justify the main driving forces of the medium term forecast to the general public and to policymakers as simple as possible. Having in mind this, it is much easier to communicate and to explain the future inflation development by the dynamics and contributions of its price components. Therefore, decomposition of projected total inflation on medium term to individual price components is also important in the process of monetary policy decision making. NBRM projection team has putted effort to make inflation decomposition model as additional set of projection information and this study is trying to explain the model of decomposition of medium term inflation obtained by

MAKPAM. With other words, this model links projected inflation by MAKPAM model with the future path of CPI components, which is the main contribution of this framework for inflation forecast's analysis. The model was actively used since April 2016 projection rounds[1].

The framework for inflation decomposition is based on the framework that is used in Czech National bank (CNB, 2015), and adapted for our country specifics. The model for inflation decomposition is small structural model, which decomposes the headline inflation on three main CPI subgroups – food prices, energy prices and core inflation. This model by structure is very similar to the small structural model which is used by NBRM for short term forecasting. The small structural model for short term projection defines the overall inflation as weighted sum of three sub-components – energy inflation, food inflation and core inflation, which are modeled by separate behavioral equations (Petrovska et al., 2017). Similarly, the model for decomposition of medium term inflation projection is consisted of behavioral equations for food, domestic oil prices, core inflation of goods and services, and identity equations for energy and total core inflation, as well as equation for total inflation (weighted sum of food, energy and core inflation). Unlike the structural model for short term forecasting, where the approach is bottom-up, in this case the approach is top-down. This framework is not a rival model to short term forecasts; moreover, the comparison of decomposed inflation parts with their near term forecasts should be an integral part of the projection process, as it is in CNB (CNB, 2015).

The model is defined as state space model, using Kalman filter, where the total inflation is given as observable variable. The model is actually filtering the future dynamic of each component through information that are contained in exogenous variables and according to total inflation dynamic. More precisely, we are trying to distribute the part of total inflation that is not explained by determinants in the component equations to the dynamics of each of the components.

Additional feature of this framework is that it provides comparison of projected inflation and its decomposition between two different rounds of projections, as well as comparison between baseline and shock scenario. Although, this is integral part of projection process, it will not be presented here.

The paper is organized as follows. Description of state space framework and Kalman filter is given in the next section. In section 3 the main equations of the model for inflation

[1] NBRM conducts two rounds of projections per year, in April and in October.

decomposition at the NBRM are explained, while the data and the results, as well as model performance are exposed in section 4. Finally section 5 concludes.

2. State Space Models and the Kalman Filter

The Kalman filter is a cast of state space models, which is useful framework in the modelling. Among its uses, it is useful tool for unobserved components modelling, such as output gap, NAIRU and time varying parameters, which are at the same time unobservable and time varying. The method that can be used to estimate the unobservable variables (`the state') given all observable data we have is known as the Kalman filter. The filter has its origin in Kalman's paper (1960), who describes a recursive solution to the linear filtering problem of discrete data.

The linear state-space model postulates that an observed time series is a linear function of a (generally unobserved) state vector and the law of motion for the state vector is first-order vector autoregression. The general linear state space form applies to a multivariate time series, y_t, containing N elements, that are observed variables at date *t*. These observable variables can be described in terms of possibly unobserved m × 1 vector, β_t, known as the state vector (Hamilton, 1994, Harvey, 1998). Let *F* and *H* be $m \times m$ and $N \times m$ matrices of constants. We assume that y_t and β_t are generated by:

$$y_t = H\beta_t + e_t,\;\; var(e_t) = R \tag{1}$$

$$\beta_t = \mu + F\beta_{t-1} + v_t,\; var(v_t) = Q \tag{2}$$

where β_t is a vector of stochastic or unobservable variables (states) and y_t is a vector of measurement variables (observed data). e_t and v_t are measurement error/ structural shock, that are white-noise processes, independent of each other with mean zero ($e_t \sim N(0,R)$, $v_t \sim N(0,Q)$, and $cov(e_t, v_t = 0)$), where matrices $R = E(e_t e_t')$ and $Q = E(v_t v_t')$. The specification of the state space system is completed by assuming that the initial state vector, β_0 has a mean of β_0 and a covariance matrix P_0, ($E(\beta_0) = \beta_0$ and $Var(\beta_0) = P_0$).

Equation 1 is called the "measurement" or "signal" equation, while equation 2 is called the "transition" or "state" equation. The measurement equation describes the relationship between observed and unobserved (state) variables, and the state equation describes the dynamics of the unobserved variables over time. The assumption for first-order autoregression in transition equation is not restrictive, since as any AR(p) process can always be re-written in first order companion form (Blake and Mumtaz, 2012).

The complete estimation procedure with Kalman filter includes: formulation of model in state-space form and given set of initial parameters, the model prediction errors are generated from the filter. These are then used to recursively evaluate the likelihood function until it is maximized (Jalles, 2009). Actually, the Kalman filter is a recursive algorithm for producing optimal linear forecasts of β_{t+1} and y_{t+1} conditional on an information set (observed data-Y_t), assuming that *H*, *F*, *R* and *Q* are known. The Kalman filter assumes that the transition and measurement equations are linear and the shocks to the system, e_t and v_t, as well as the initial state, are all normally distributed (Gaussian). The full representation of the Kalman filter algorithm is given in Appendix.

3. Model Structure and Core Equations

The model for inflation decomposition is a small structural model with several behavioral equations, identities, definitions and calculations. It is a system of linear equations. The model decomposes total inflation on three main sub groups: food, energy and core inflation. Unlike food inflation, for which there is one equation, Energy and Core inflation are weighted sum of their components. Energy component has three sub components: domestic oil derivatives prices (petroleum), electricity and heating. As the electricity and heating components are regulated prices, there are no behavioral equations for them, only for petroleum prices. Core inflation is composed of goods and services, and for each of them there is behavioral equation. Figure 1 presents the "price tree".

The food inflation (Δcpi_t^{food}) is assumed to be determined by inertia, lag of domestic petroleum prices, domestic output gap, lag of world grain food prices and foreign effective food prices:

$$\Delta cpi_t^{food} = \mathrm{f}(\Delta cpi_{t-1}^{food}, \Delta cpi_{t-1}^{petroleum}, Y\,gap_t, \Delta grain\_index_{t-1}, cpi\_ef_t^{foriegn\,food}) \tag{3}$$

where $cpi_t^{petroleum}$ stands for domestic oil derivatives prices, $Y\,gap_t$ is domestic output gap, $grain\_index_t$ is composed index of world wheat and maize prices, $cpi\_ef_t^{foriegn\,food}$ is foreign effective food index. The operator Δ is a first (quarter) difference operator.

The equation describes that food inflation is determined by domestic factors and import prices. Output gap represents the impact from the domestic demand, and an increase in domestic demand will increase food prices. Domestic prices of oil derivatives are included as cost factor, which means that rise in oil prices will put pressure on production costs and to the final food prices. Grain index represents the import prices of raw food, while the foreign effective food index represents the import prices of final products. Both import prices have positive effect on domestic food prices.

The domestic oil derivatives prices ($\Delta cpi_t^{petroleum}$) are determined by inertia and world oil prices (current and first lag):

$$\Delta cpi_t^{petroleum} = \mathrm{f}(\Delta cpi_{t-1}^{petroleum}, \Delta oil\_index_t, \Delta oil\_index_{t-1}) \tag{4}$$

where oil_index_t stands for world oil prices. Domestic prices of oil derivatives are determined by Regulatory energy commission (REC), which makes price's adjustment every two weeks according to world prices and exchange rate MKD/USD in the previous two-week period. Hence, current world oil prices value have the bigger coefficient than the lagged one.

The core inflation, which in our case is the headline inflation excluding food and energy components captures lower frequency changes in the general price level. For core inflation or "underline inflation" we adopt the approach of using separate models for core goods and core services to explain the dynamics of aggregate core inflation. Domestic factors are seen as those factors that influencing services inflation primarily, while global factors play a larger role in the goods inflation process (Peach et al., 2013).

The equations for core good and services prices ($\Delta cpi_t^{core_g}$and $\Delta cpi_t^{core\_s}$) are very similar, but still different, and are given in equations 5 and 6, accordingly:

$$\Delta cpi_t^{core\_g} = \mathrm{f}(\Delta cpi_{t-1}^{core\_g}, Y\ gap_t, \Delta cpi_{t-3}^{energy}, \Delta cpi\_ef_{t-2}^{foriegn}) \tag{5}$$

$$\Delta cpi_t^{core\_s} = \mathrm{f}(\Delta cpi_{t-1}^{core\_s}, Y\ gap_{t-1}, \Delta cpi_{t-3}^{energy}) \tag{6}$$

where cpi_t^{energy} is domestic energy price index and $cpi\_ef_t^{foriegn}$ is foreign effective inflation index. In standard models of Philips curve for core inflation forecasting, one of the main explanatory variable is activity gap (output gap or unemployment gap). Activity gap variable measures the resource utilization, or the extent of excess demand or slack in an economy. The link between excess demand and inflation does not apply in the same way to the parts of the CPI that change for exogenous reasons. Hence, the activity gap is a kind of measure for domestic pressures on prices. In our case, output gap is included as a proxy for domestic demand. Other variable is the domestic energy index, which is included with a three-quarter lag in order to capture the indirect effect of energy prices changes, as firms gradually pass on their increased costs of production in the form of higher prices for final goods or services. One of the main differences between the two core equations is that core goods inflation has one additional factor, which is the foreign effective inflation index. As it was already mentioned, it is considered that reasonable part of the goods is imported and foreign prices have also effect on domestic goods prices excluding food and energy (Peach et al., 2013). The lagged core inflation represents intrinsic dynamics, that is the effects of things like contractual lags or other costs of adjustment that lead to stickiness in prices, even in the absence of expectations lags (Benes and N'Diaye, 2004).

Additionally to these behavioral equations, the electricity and heating prices are determined as sum of prices with no change assumed (the index is kept same as the last known value for the whole period of projection, i.e. zero q-o-q change) plus regulatory change:

$$\Delta cpi_t^{electricity\_tot} = \Delta cpi_t^{electricity} + ADM_t^e \tag{7}$$

$$\Delta cpi_t^{heating\_tot} = \Delta cpi_t^{heating} + ADM_t^h \tag{8}$$

where ADM are series that capture the regulatory change if some regulatory decision regarding these prices is known in advance, or it will take 0 otherwise.

The identity equations regard to Energy prices (Δcpi_t^{energy}), Core Inflation (Δcpi_t^{core}) and Total inflation (Δcpi_t^{total}), which are weighted sum of their sub-components:

$$\Delta cpi_t^{energy} = w^{petroleum} * \Delta cpi_t^{petroleum} + w^{electricty} * \Delta cpi_t^{electricity\_tot} + w^{heating\_tot} * \Delta cpi_t^{heating} / \left(w^{petroleum} + w^{electricty} + w^{heating}\right) \tag{9}$$

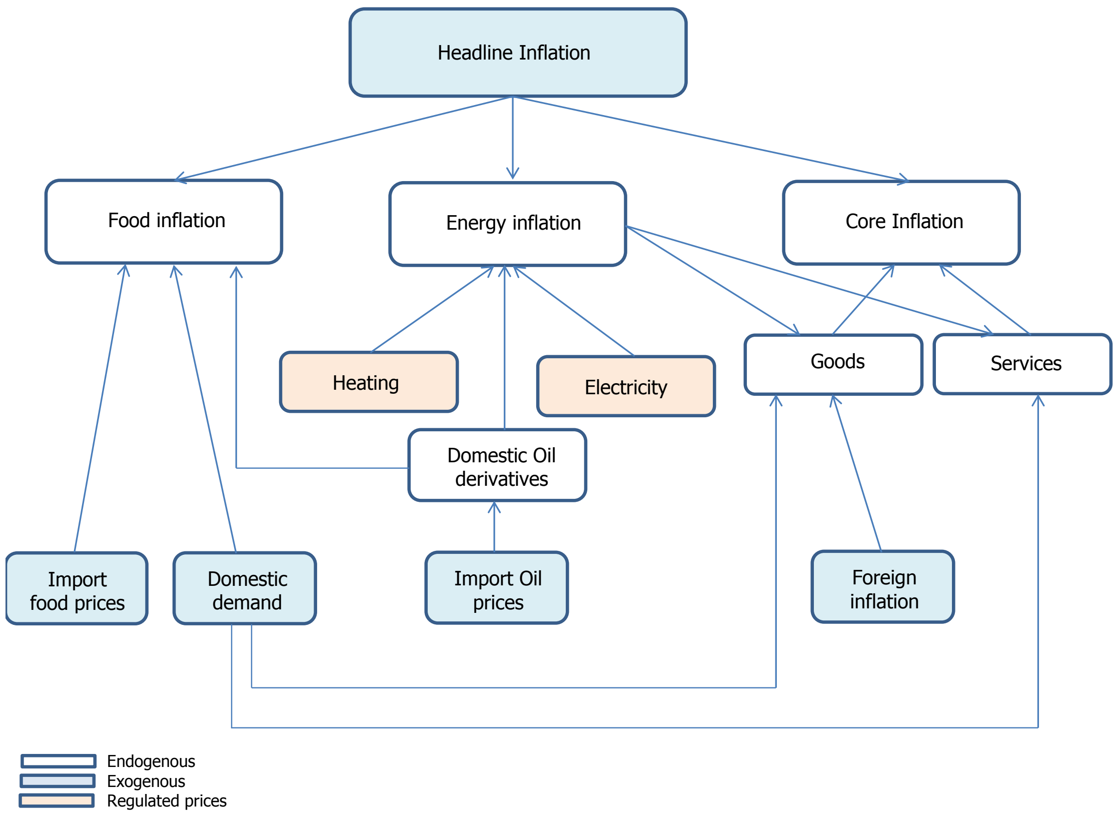
$$\Delta cpi_t^{core} = w^{core\_g} * \Delta cpi_t^{core_g} + w^{core_s} * \Delta cpi_t^{core_s} / (w^{core_g} + w^{core_s}) \tag{10}$$

$$\Delta cpi_t^{total} = w^{food} * \Delta cpi_t^{food} + w^{energy} * \Delta cpi_t^{energy} + w^{core} * \Delta cpi_t^{core} \tag{11}$$

where elements marked with w are weights of each component to total CPI and the weights of energy and core inflation are sum of the weight of their sub-components.

Figure 1

CPI components ("price tree")



The Kalman filter is used for estimation of separate components of inflation (food inflation, domestic oil derivatives prices, core inflation of goods and of services), where inflation components are filtered as unobserved variables at forecast. All other exogenous variables are treated as observed variables (headline inflation obtained by MAKPAM model, domestic output gap, exchange rate, oil prices, wheat prices and maize prices, foreign effective inflation and foreign effective food prices).

The parameters in the model are calibrated. The process of calibration is based on combination of expert judgments, individual estimations of the variables and trial and error process, taking into account country specific behavior of domestic consumers.

4. Data and Results

4.1. *Data*

The dataset is on quarterly basis, starting from 2003. The choice of this starting sample point has been dictated by statistical database, as inflation sub components by COICOP classification start from January 2003. Domestic components price indices and the weights of all sub-components are according to State Statistical Office of Republic of Macedonia (SSO) with base year 2016=100[2] and they are seasonally adjusted. Domestic Output gap and total inflation are taken as output of MAKPAM model (NBRM Projection April 2017). All foreign prices are expressed in Euros, as Macedonia is a country with *de facto* fixed exchange rate to the Euro. World oil, wheat and maize prices are taken from IMF Prices commodity database, while the Exchange rate USD\EUR is from ECB - Euro foreign exchange reference rates database[3]; Foreign effective inflation index and Foreign effective food index are calculated by NBRM staff as weighted sum of the most significant trade partners for import of consumption goods and food[4], accordingly, for which Eurostat data is used. For the projection period, various international projections are used such as Consensus Forecast or IMF WEO database[5].

4.2. Results

For the purpose of this study and presentation of model results, NBRM Projections of April 2017 data are used. According to MAKPAM projections, the inflation is expected to increase gradually by 1.3% and 2.1% in 2017 and 2018, accordingly. This dynamic is given exogenous to the decomposition model. Additionally, for the regulated prices – heating and electricity prices, which are part of energy component, it is assumed that there will be no changes in the level prices in

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[2] The indices and the weights of domestic inflation and its components are subject of change each year by SSO, taking last year as a base.

[3] For the purposes of April 2017 projection, IMF Prices commodity database with last available data of February 2017 is used, while for the exchange rate ECB database, the data available up to 23.03.2017 are used.

[4] The Foreign effective inflation index includes Bulgaria, Germany, Greece, France, Italy, Austria, Slovenia, Croatia and Serbia, while the Foreign effective food index includes Bulgaria, Germany, Greece, Austria, Croatia and Serbia. The weight structure is based on the normalized shares of their average shares in the period 2013Q1-2016Q3.

[5] For the purposes of April 2017 projection, Consensus Forecast and Eastern Europe Consensus Forecasts, March 2017 and WEO October 2016 are used.

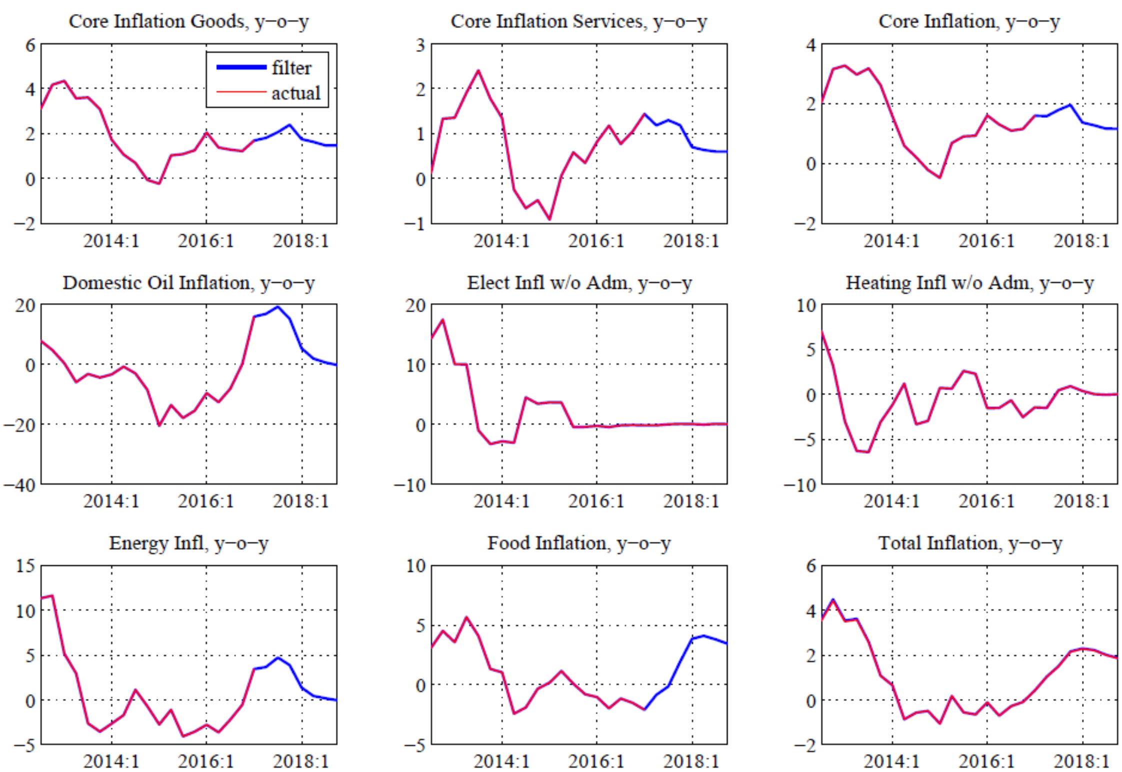
the projection period[6], due to absence of any announcement for future changes by the Regulatory energy comission. Figure 2 shows the annual growth rates of total inflation and projected (filtered) dynamics of its sub components.

The decomposition of total inflation shows negative food inflation in the first year of projection and solid recovery in the next one. Domestic oil derivatives prices will grow in 2017, as well as in 2018, but with slower pace. Core inflation will continue to grow in 2017, but it will slow down in 2018, the dynamics that show both of its sub categories (goods and services). The decomposition of total inflation is very similar to the projections obtained from near term forecast.

Figure 2

Annual growth rates of total inflations and its components

(in %)



4.3. Model performance

4.3.1. Factor explanation

[6] The assumption for no change in the level prices in the projection period implies only 0% q-o-q change, but still there might be some price changes on annual base in the first year of projection as result of base effect.

In this section model performances will be presented, more specifically how well the endogenous variables in the model given in Section 3 are explained by their determinants and what are the contributions of those determinants in the dynamics of each inflation component. This is easily answered by simple decomposition of inflation components, taking into account the data, the equations and calibrated coefficients. The decomposition analysis for past data will show us the fit of the model, while the decomposition for period of projections will give us the true meaning of this exercise. In other words, it will show us, to which extend the projected dynamics of each inflation sub components is explained by exogenous factors' projections and by the projected dynamics of total inflation, which is distributed among components, with exception of regulated prices. The distributed part is transferred into component's dynamics through equation shocks. Two aspects of decomposition are considered: (i) the total effect of each inflation sub component; and (ii) the true contribution of each sub component that is determined only by its factors included in their equations. The last decomposition will show us roughly how much of the projected dynamics of total inflation, that is given to the model, is not explained by factors included in the model and should be redistributed in the dynamics of the sub components. The distributed part is different in each process of projection, depending on model setting, projected total inflation and projected path for other exogenous factors, thus the results presented here are just an illustration how the model is functioning.

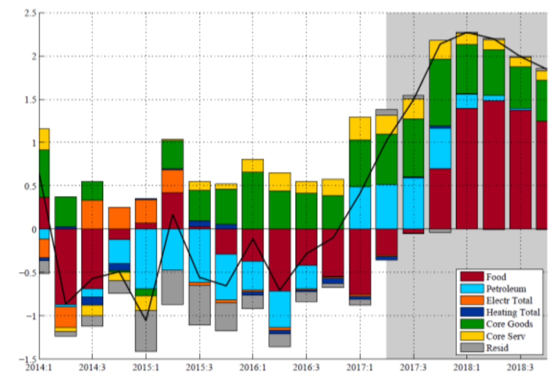
The first two panels in Figure 3 (A and B) are showing the decomposition of total inflation on its sub components. The analysis will be focused on only of the period of projection. Analyzing Panel B, which is decomposition of inflation on more aggregate components, it shows that Energy and Core components have positive contribution through whole projection period, while food inflation has a small negative effect in 2017, while in the later period food has dominant positive contribution. As it can be seen, contribution of all components sum up to total inflation. Unlike this, in the graph in panel C is shown the contribution of the components explained only by their determinants in each of the equations in the model as well as the effect of regulated prices. The rest, marked with grey, represents the unexplained part of the total inflation that is distributed in the inflation sub components dynamics by Kalman filter, except regulated prices.

Figure 3

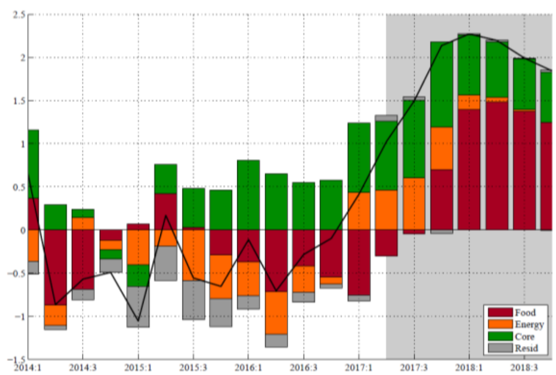
Headline inflation (y-o-y) and the contribution of the sub components

(in % and in p.p.)

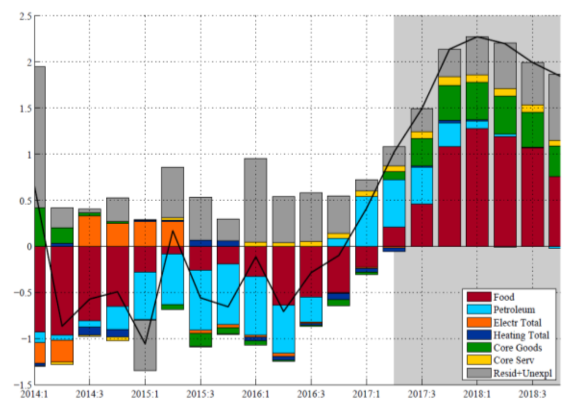
A)



B)



C)



In the following part, it will be shown the decomposition of individual equations, where historical data decomposition will give the goodness of the equation fit, while the decomposition in the period of projections will give the answer about contribution of determinants as well as the distributed part, captured through the residuals. Figure 4 shows the decomposition of each inflation component equation, as well as the identities, such as Energy and Core inflation.

The decomposition of food inflation shows that its determinants explain most of the dynamics in the past, which point to good fit of the equation. As it can be seen, the world grain food prices have the biggest contribution in most of the time, while domestic oil prices have significant negative effect starting from second half of 2015, which is expected as the oil prices had a significant fall from 2014Q4 onwards (see panel B). In the period of projection, beside the previous two factors, the food inflation is explained additionally by foreign food inflation, while output gap has a small positive effect. The shock (redistributed part) also has solid contributing effect[7], which adds up to the total dynamics of food inflation.

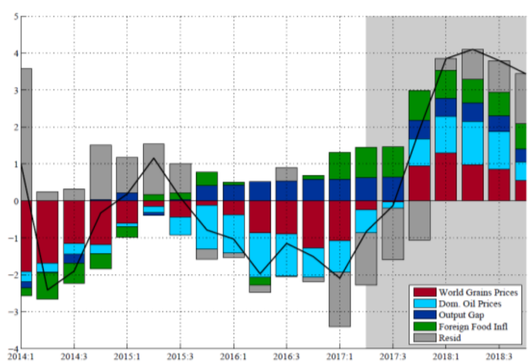
Domestic oil inflation, which is determined only by world oil prices, has a fairly good fit for the historical data. This is expected, as it was already explained in Section 3, domestic oil prices are set on regular basis, every two weeks, in accordance of world prices and exchange rate developments in previous two-week period. Apart from world prices, redistributed effect from total inflation dynamics is also present in projected dynamics, contributing additionally in domestic oil prices growth.

Figure 4

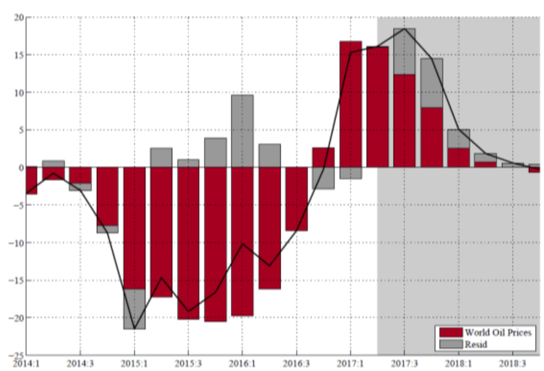
Various domestic inflations (y-o-y) and contributions of its determinants

(in % and in p.p.)

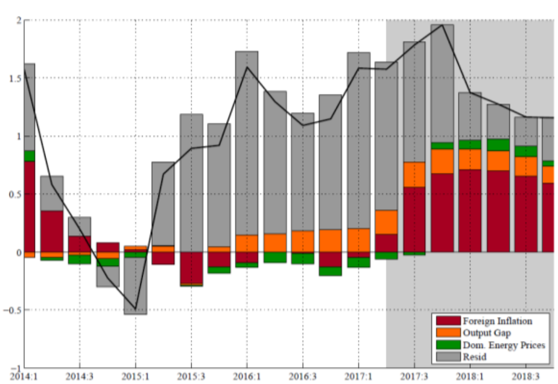
A) Food inflation



B) Domestic oil inflation

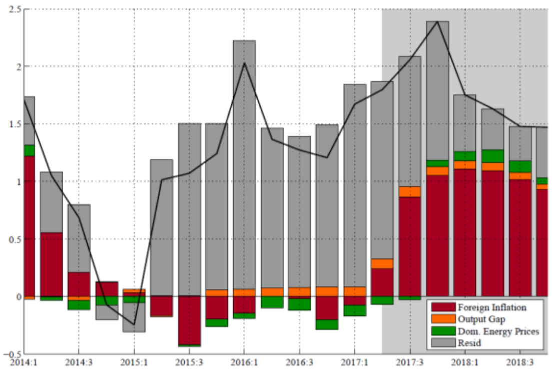


C) Core inflation

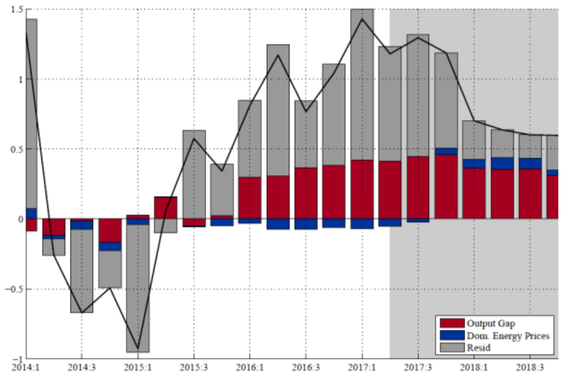


[7] Part of the shocks in the begging of the projection period is due to inertia (the measurement error in the past); it is not the true effect of redistribution of total inflation dynamics. This is valid for all core equations in the model, as they all include inertia.

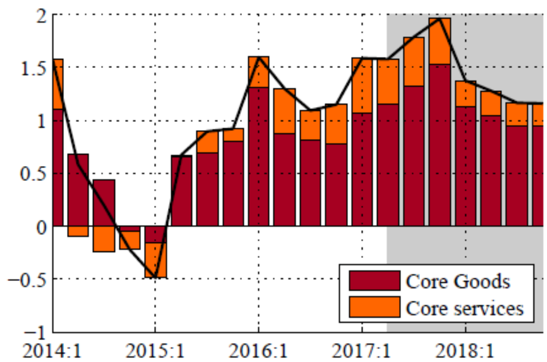
D) Core inflation - Goods



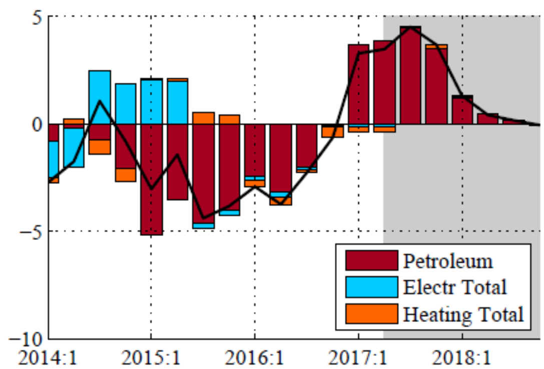
E) Core inflation - Services



F) Core inflation



G) Energy inflation



The figures in panels C-E refer to Core inflation and its sub components. As it was already mentioned, the aggregate core inflation was divided in two parts: goods and services. The both core inflations don't have a satisfactory fit; the determinants don't explain much of their dynamics, particularly for goods. As it can be seen on Figure 4 in Panel D, goods core inflation it is partly determined by import prices at the beginning of the analyzed period, while in the later period, only domestic demand, albeit very weak, contributes to price's growth. The explanatory power of domestic demand is higher in case of services core inflation (Panel E), which is expected, as services are mostly driven by domestic factors. To this end, aggregate core inflation decomposition (Panel C) is very similar to that of goods, as goods consists almost 2/3 of total core inflation. The weak explanatory power of the determinants of core inflation to some extent lay in the fact that some of the prices developments were not related with economic factors, but were determined by one time factors, such as excise duty on alcohol and tobacco, changes in some other administered prices[8] or effects of changes in the process of inflation measurement[9] etc. Regarding projections of core inflation, all factors contribute positively, except for negative

[8] Such as tuitions, fees for administrative documents etc.
[9] Change in the type of the product that was followed for prices collection.

effect of the domestic energy prices in 2017, while the redistributed part of total inflation dynamics also has positive contribution in the future dynamics, with stronger effect in 2017[10] and smaller in 2018.

The identities, Core and Energy inflation, are the weighted sum of its components. The projected core inflation is mostly driven by goods' component, although services also have solid contribution. Energy inflation is almost completely determined by the projected dynamics of domestic oil prices.

4.3.2. In-sample forecast

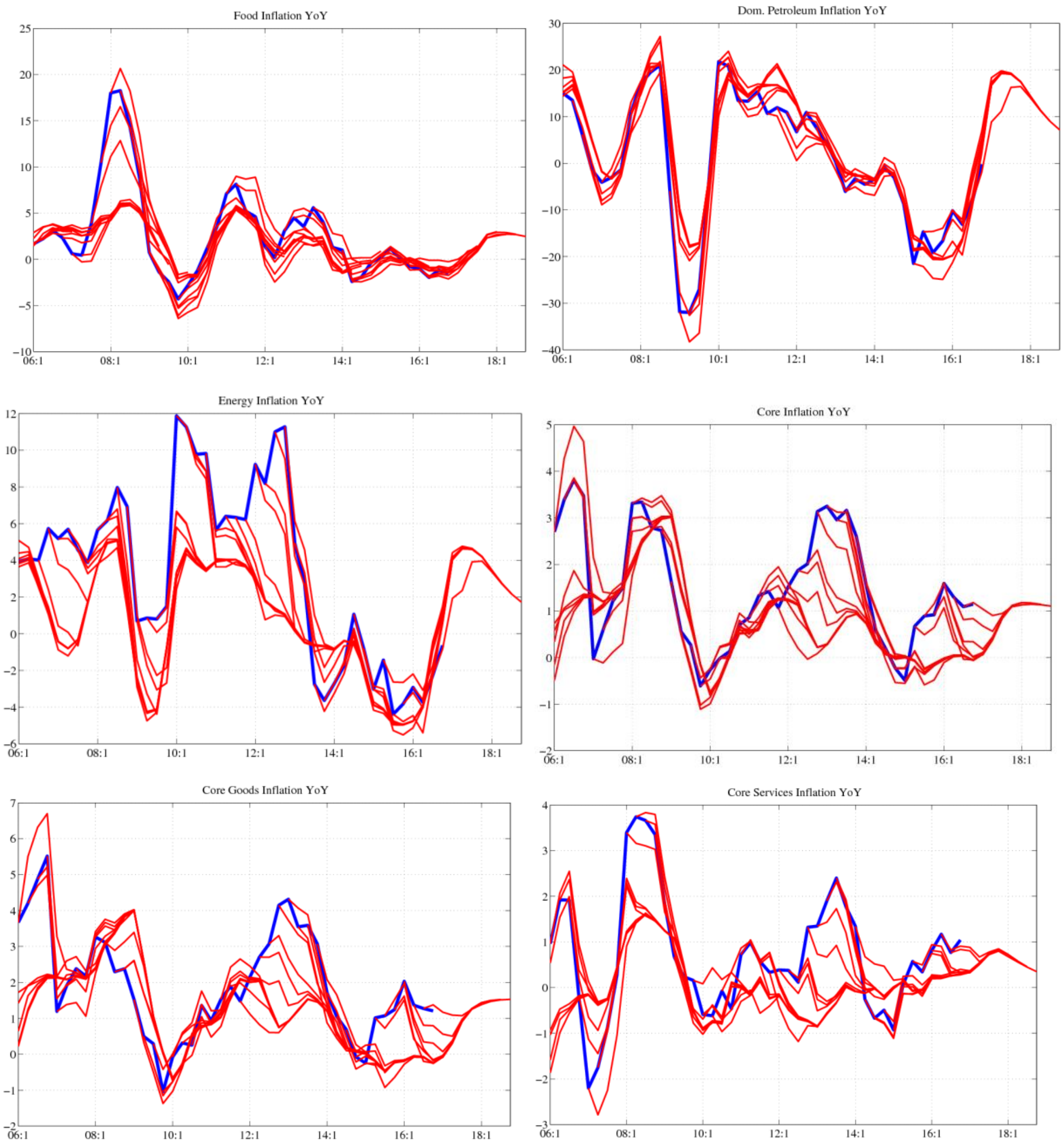
The forecasting properties, shown in previous section, are evaluated using the regular fully-fledged forecasts since Q2 2017. Other option for model performance check is in-sample forecasting, which is the recursive filtering and forecasting exercises. In principle, the iterations are based on the regular forecast scheme, with assumption that all exogenous variables are known *ex ante* for the whole horizon and without imposing any expert judgment. Each time, a filter is run, given the available data, and the forecast is simulated. Then the sample is extended by one period and the forecasting process is repeated until the last observable data is available. The final result of the in-sample analysis consists of a large number of mechanical model simulations.

Figure 5 presents an example of recursive filtering and forecasts of inflation components. The mechanical structure and absence of conditioning makes these forecasts different from the regular process of forecasts. Actually, according to Andrle et al. (2009), this exercise of in sample simulations is more an illustration of the procedure than a measure of the model's dynamic and forecasting properties. Nevertheless, the blue line represents the actual historical data, while red lines are 8 quarters ahead in-sample forecasts at various points in the past.

[10] See footnote 8.

Figure 5

Recursive forecasts (In sample forecast)



The results of in sample analysis confirm the conclusions of the decomposition analysis. Food and domestic oil prices have a relatively good fit, as the in-sample forecast is generally close to actual outcomes and it captures turning points quite well. Although domestic oil prices are forecasted well in most of the time, still energy prices don't share the same outcome. The deviation of in sample simulations from actual data is the result of regulated prices that are not controllable in this exercise as in the regular forecast process, where they are actually determined *a priori*. In-sample forecasts for core inflation components and for aggregate core inflation show mixed goodness of fit. There are relatively better forecasts up to 2012, while in the following period the simulations are not so good in capturing the turning points and mostly show undershooting compared with the actual outcome. Overall, the results indicate that the forecasting performance of the model is good enough for the model to be a useful analytical tool in the process of inflation forecast.

5. Conclusions

This paper has developed a methodology for decomposition of total inflation, obtained with macroeconomic model (MAKPAM) of NBRM, to CPI components. This decomposition of inflation to its components - food, energy and core inflation can facilitate discussion on inflation and help monetary policy decision making. Our methodology uses a small system that accounts for the interactions among total inflation dynamics and other exogenous factors, such as output gap, world commodity prices, and foreign effective inflation, to obtain projections for inflation components by applying the Kalman filter procedure.

We applied the model to Macedonian data, and adapted it in accordance with country specifics. We have introduced the main characteristics of the model and focused mainly on forecasting performances of the model. The results of CPI component projections are in line with the expectations, and we believe that obtained filtrations of inflation components are realistic and provide a solid base for a deeper analysis of inflation. It was also shown by decomposition analysis and by performing an in sample forecast exercise, that most of our estimates, except for the core inflation components, give satisfactory results. Namely, food and domestic oil prices have a relatively good fit, and the recursive forecasting shows that the capture of turning points is quite well. Regarding the core inflation, there is weak explanatory power of the determinants after 2012, which to some extent is effect of one time factors rather than economic factors. Overall, the performances tests showed that the model is useful analytical tool in the process of inflation forecast.

Nevertheless, there is still room for improvement and future work on this model. One point of improvement is including forward-looking inflation expectations in some of the core equations, which is in accordance with the modern macroeconomic theories.

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Appendix

Kalman filter

The Kalman filter operates within a state-space representation. The linear state-space model postulates that an observed time series is a linear function of a (generally unobserved) state vector and the law of motion for the state vector is first-order vector autoregression, given in equation 1 and 2:

$$y_t = H\beta_t + e_t,\ \ var(e_t) = R \tag{1}$$

$$\beta_t = \mu + F\beta_{t-1} + v_t,\ var(v_t) = Q \tag{2}$$

where β_t is a vector of stochastic or unobservable variables (states) and y_t is a vector of measurement variables (observed data); e_t and v_t are measurement error/ structural shock, that are white-noise processes, independent of each other and of the initial value β_0 with mean zero ($e_t \sim N(0, R)$, $v_t \sim N(0, Q)$, and $cov(e_t, v_t = 0)$), where $R = E(e_t e_t')$ and $Q = E(v_t v_t')$; *F* and *H* are $m \times m$ and $N \times m$ matrices of constants. The specification is completed by assuming that the initial state vector, β_0 has a mean of β_0 and a covariance matrix P_0, ($E(\beta_0) = \beta_0$ and $Var(\beta_0) = P_0$).

In state-space representation, Equation 1 is called the "measurement" or "signal" equation, while equation 2 is called the "transition" or "state" equation.

The Kalman filter is a recursive algorithm for producing optimal linear forecasts of β_{t+1} and y_{t+1} conditional on an information set (observed data-Y_t), assuming that *H*, *F*, *R* and *Q* are known. The Kalman filter assumes that the transition and measurement equations are linear and the shocks to the system, e_t and v_t, as well as the initial state, are all normally distributed (Gaussian). Because a normal distribution is characterised by its first two moments, the Kalman filter can be interpreted as updating the mean (β_t) and variance-covariance matrix (P_t) of the conditional distribution of the state vector as new observations become available. When the normality assumption is dropped, the Kalman filter is still optimal estimator, as it minimises the mean square error within the class of all linear estimators (Harvey, 2006).

The Kalman filter provides solution of three types of estimation problems (Kalman, 1960; CNB, 2015):

- Filtering - estimation of β_t using information up to time t $\{y_s\}_{s=1}^{t}$, which aims to update our knowledge of the system in general and that of the state;

- Smoothing – estimation of β_t using information of the whole sample $\{y_s\}_{s=1}^{T} (T \geq t)$ [11];
- Prediction – estimation of β_{T+h} given the information $\{y_s\}_{s=1}^{T}$, which aims to forecast β_{T+h} or $y_{T+h}$for $h > 0$.

Single iteration of Kalman filter consists of two steps: prediction and update. The initialization of the Kalman filter is starting from initial condition, which can be specified as β_0 and P_0 or $\beta_{1/0}$ and $P_{1/0}$. Given these initial conditions, the Kalman filter delivers the optimal estimator of the state vector as each new observation becomes available. When all observations have been processed, the filter yields the optimal estimator of the current state vector, and/or the state vector in the next time period, based on the full information set.

The filter evaluates following equations recursively:

$\beta_{t|t-1} = \mu + F\beta_{t-1|t-1}$ (Prediction of state β) (3)

$P_{t|t-1} = FP_{t-1|t-1}F' + Q$ (Prediction of P – covariance matrix of predicted state) (4)

$y_{t|t-1} = H\beta_{t|t-1}$ (Prediction of observables y_t) (5)

$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - H\beta_{t|t-1}$ (Prediction error) (6)

$f_{t|t-1} = HP_{t|t-1}H' + R$ (Variance of predicted error) (7)

$K_t = P_{t|t-1}H'f_{t|t-1}^{-1}$ or $K_t = P_{t|t-1}H'(HP_{t|t-1}H' + R)^{-1}$ (Kalman gain) (8)

$\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1}$ or $\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1}H'f_{t|t-1}^{-1}\eta_{t|t-1}$ (Update of β) (9)

$P_{t|t} = P_{t|t-1} - P_{t|t-1}H'f_{t|t-1}^{-1}HP_{t|t-1}$ or $P_{t|t} = (1 - K_tH)P_{t|t-1}$ (Update of P) (10)

K_t is known as Kalman gain and it shows how the new information (a prediction surprise) is reflected in the update of state estimation or the weight attached to new information (about the state) contained in the prediction error.

Taken together the equations 3 to 10 make up the Kalman filter.

In the case of smoothing, the estimates of β_t are based on observation available for t=0,…T, using the notation $\beta_{t|T}$. Unlike the Kalman filter, the smoother estimate final state using all variable information, but also taking into account previous estimates. It is backwards recursions to update the filtered estimates. The Kalman smoother is represented and recursively run as follows:

[11] The Kalman filter is a one-sided, causal estimate of the state β_t based on information up to the period [t_0,….,t]. The Kalman smoother is a two-sided, non-causal filter that uses all available information to estimate the state $\beta_{t/T}$ based on [t_0,….,T] (Andrle, 2013).

$$\beta_{t|T} = \beta_{t|t} + C_t(\beta_{t+1|T} - \beta_{t+1|t}) \quad (11)$$

$$P_{t|T} = \beta P_{t|t} + C_t(P_{t+1|T} - \beta_{t+1|t})\, C_t{}' \quad (12)$$

$$C_t = P_{t|t}F(P_{t+1|t})^{-1} \quad (13)$$

where the initial condition is $\beta_{T|T}$.

Running the Kalman filter up to time T gives the current estimate of the state vector. In addition it gives $\beta_{T+1|T}$ and the one-step-ahead predictor$y_{t+1|T}$. With initial value $\beta_{T|T} = \beta_T$, the multi-step predictor of state can be written as:

$$\beta_{T+h|T} = \mu + F\beta_{T-1+h|T} \quad (14)$$

Similarly, with $P_{T|T} = P_T$, it gives:

$$P_{T+h|T} = FP_{T+h|T}F' + Q \quad (15)$$

Thus $\beta_{T+h|T}$ and $P_{T+h|T}$ are evaluated by repeatedly applying Kalman filter prediction equations. The minimum mean square error (MMSE) of y_{T+h} can be obtained directly from $\beta_{T+h|T}$. Taking conditional expectations in the measurement equation for y_{T+h} gives:

$$E(y_{T+h}|Y_T) = \tilde{y}_{T+h|T} = H\beta_{T+h|T} \quad (16)$$

with MSE matrix:

$$MSE(\tilde{y}_{T+h|T}) = f_{T+h|T} = HP_{T+h|T}H' + R \quad (17)$$

When the normality assumption is relaxed, $\beta_{T+h|T}$ and $\tilde{y}_{T+h|T}$ are still minimum mean square linear estimators (Harvey, 2006).