National Bank of the Republic of Macedonia

Research Paper

2015

Kalman Filter Estimation of the Unrecorded Economy in Macedonia

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Abstract

This paper estimates the unrecorded economic activity and employment in Macedonia during 1998-2013. We model the unrecorded economic activity as an unobserved process that depends on tax "burden", business regulations and quality of governance, and affects cash in circulation, foreign currency in circulation and energy consumption. We then estimate it, using the Kalman filter. After we estimate the unrecorded economic activity in this way, we calculate the unrecorded employment using a wide range of values for the employment/economic activity ratio. Finally, after we estimate the unrecorded employment, we will be able to calculate the true rate of unemployment in Macedonia. The findings suggest that unrecorded economic activity in Macedonia has declined sizeably, from 34 percent of official GDP in the late 1990's, to 10 percent in 2010's. Unrecorded employment has declined over this period, too, from 160-200,000 in late 1990's to 70-90,000 currently. Accordingly, the true rate of unemployment was likely to be around 20-22% in 2013, which is some 7-8 percentage points below the official one.

JEL Classification: E26, J46,

Keywords: unrecorded economy, unrecorded employment, Kalman filter, Macedonia

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I.   Introduction

The aim of this paper is to estimate the unrecorded economy in Macedonia, for the period 1998-2013. What we name "unrecorded economy" can be met in the literature under various names, such as unofficial, undeclared, grey, shadow, hidden, informal economy. Though there may be some differences between each of these, they refer to essentially the same thing - economic activity which is not included in the official figures.

It is important to understand the unrecorded economy simply because it affects many segments of the economic and social environment. People working in it do not enjoy the work-related rights and benefits they should, such as health insurance, pensions, vacation, etc. Companies that constitute it make an unfair competition to companies that work in the formal sector because of the lower taxes they pay (due to tax evasion). These unpaid taxes then represent foregone revenues for the governments, which could be used for schools, hospitals, roads and the like. Finally, central banks should also care for the unrecorded economy, because inflation, the main object of interest of central banks, may be affected by the unrecorded economy.

Different ways of estimating unrecorded economy exist in the literature. First, direct methods, which try to estimate it using surveys (Isachsen et al, 1982, Morgensen et al, 1995). Then, there are the indirect methods, which try to approximate it through some data on the aggregate economy, such as cash in circulation (currency method, first applied by Tanzi, 1980) or the energy use (energy consumption method, first proposed by Lizzeri, 1979). Very popular approach from the indirect methods is the MIMIC (Multiple Indicator Multiple Causes) approach, pioneered by Weck (1983) which models the unrecorded economy as a latent variable, which depends on several causes and is reflected in several indicators.

In this paper, we adopt an approach that follows the same intuition as the MIMIC approach, but overcomes some of its main drawbacks. Differently from the MIMIC approach, it does not model the unrecorded economy as a *latent* variable, but only as *unobserved*. Furthermore, it produces a *direct estimate of the size* of the unrecorded economy, differently from the MIMIC approach, which produces only an *index* of it, which then has to be transformed into cardinal values assuming some value for the unrecorded economy for some period. Our approach is based on a Kalman filter estimation of a MIMIC-type model of the unrecorded economy, in which the unrecorded economy is a function of the tax "burden", the business regulation and the quality

of the governance, and is reflected in the cash in circulation, the foreign currency in circulation and the energy consumption. The foreign currency in circulation is an important extension to the existing literature, because in small open economies such as Macedonia, unrecorded transactions are often carried out in foreign currency (usually euro), instead of the national currency.

After we estimate the unrecorded economic activity in this way, we approximate the number of people engaged in it, by multiplying the unrecorded activity by the number of employees per unit of formal activity from the official data (employees/GDP). We use a wide range of values for the employees/GDP ratio, obtained from the sectors where the unrecorded economy is usually prevalent, in order to account for the uncertainty regarding the structure of the unrecorded economy. Finally, after we estimate the unrecorded employment, we sum it with the formal employment and calculate the true rate of unemployment in Macedonia.

The rest of the paper is structured as follows. Section II discusses the definition of unrecorded economy and its determinants. Section III presents the different approaches for estimating unrecorded economy. Section IV discusses the model used in this paper. Section V presents the data, while section VI briefly explains the Kalman filter algorithm. Section VII presents the results and section VIII concludes.

II.   Unrecorded economy – definition and determinants

We define the unrecorded economy as market based economic activity, which is not included in the official figures for the Gross Domestic Product, which occurs due to tax "burden", cumbersome business regulation or lack of trust in the authorities. Unrecorded employment is then the employment which is associated with this unrecorded economic activity.

Our definition of the unrecorded economy is similar to the definitions found in the literature (see Frey and Pommerehne, 1984, Feige, 1990, Smith, 1994, European Commission, 2007, Williams and Renooy, 2008, Schneider et al., 2010). Several elements of the definition deserve explanation. First, it includes market-based activity, i.e. activity for which people pay with money. It does not include activities which are done for something in return. Second, it refers to activities which are excluded from the official estimates of the GDP, which itself includes a correction for the informal economy. Third, it refers to activities that appear as a consequence

of the taxes (including the social contributions), or the business regulation, or the government effectiveness, or the corruption. Some other potential determinants are not included. Fourth, it may include both legal and illegal activities.

The literature on the unrecorded economy has identified several potential causes. The **first** cause refers to taxes. The higher the taxes (including the social contributions) are, the higher the informal/unrecorded economy is likely to be, because people will have more incentives to work in the informal sector, where taxes are not paid or partially paid. Virtually, no study on unrecorded economy omits taxes, including Tanzi (1980), Thomas (1992), Schneider (1994), Lippert and Walker (1997), Loayza (1997, Schneider et al., 2010). The **second** group of factors relates to business regulations. If the regulations are too complex and cumbersome, people would try to avoid them and work in the informal sector instead (De Soto, 1986, Johnson et al., 1997, Loayza, 1997, Friedman et al., 2000, Dreher et al., 2009, Schneider et al., 2010). The **third** group refers to corruption. The relationship between corruption and unrecorded economy is positive. There are two main channels through which corruption can affect unrecorded economy. First, corruption is a form of tax. Hence, if corruption is high, people would tend to work in the informal sector, because there they do not have to bribe the officials (Johnson et al., 1997, Friedman et al., 2000, Choi and Thum, 2005). Second, corruption is likely to affect the enforcement of regulations. If corruption is high, enforcement is likely to be lower, i.e. it would be easier for people to stay out of the formal sector (Hindriks et al., 1999). The **fourth** group of factors refers to government effectiveness. If the government is more effective in the provision of public goods, tax morale would be higher and people would prefer to stay in the formal sector (Torgler and Schneider, 2007a, Torgler and Schneider, 2007b).

III.   Methods to estimate unrecorded economy in the existing literature

There are two main approaches for estimating the unrecorded economy. The first one is based on **direct** observation, usually through surveys. The surveys can ask firms and individuals if they compete against firms that operate informally, if they have experience with such firms, and so on. This approach has been used by Isachsen et al. (1982) and Morgensen et al. (1995). The second approach is based on **indirect** measurement, through macroeconomic data. The most common methods here are the currency demand method, the energy consumption method and the Multiple Indicators Multiple Causes (MIMIC) method. **The currency demand method** was

proposed by Tanzi (1980), who estimated the underground economy in the US during 1929-1980. This method builds on earlier work of Cagan (1958) and Gutmann (1977). It assumes that people that engage in the informal economy would prefer to use cash, in order to avoid leaving traces. The essence of the method is to regress cash in circulation (as a share of some money aggregate, like M1) on the tax rate, per capita income, the share of wages and salaries in national income and other variables that may affect the cash in circulation. Then, the residual increase in the cash/M1 ratio that cannot be explained by the conventional explanatory variables is attributed to the unrecorded economy. **The energy consumption method** was first used by Lizzeri (1979) and Del Boca and Forte (1982), but the most common version of it is associated with Kauffman and Kaliberda (1996). It is based on the assumption that unrecorded economic activity requires some physical input, similarly to the formal economic activity. Usually, the input that is used is the electricity consumption. Assuming certain elasticity for the electricity/economic activity ratio, one can attribute the difference between the actual electricity consumption and the one explained by the official GDP to the unrecorded economic activity. **The MIMIC method** has been pioneered by Weck (1983), Frey and Weck (1983a,b), Frey and Weck-Hannemann (1984), and popularized by Friedrich Schneider and co-authors (Schneider and Enste, 2000, Dell'Anno and Schneider, 2003, Bajada and Schneider, 2005, Schneider, Buehn and Montenegro, 2010, to name a few). It is based on structural equation modelling which treats unrecorded economy as a latent variable. In Schneider, Buehn and Montenegro (2010), where shadow economies are estimated for 162 countries for 1999-2007, the shadow economy is modelled as being driven by tax "burden" (share of direct taxes in total taxation, size of government and fiscal freedom index), intensity of regulations (business freedom index), public sector services (government effectiveness index) and the official economy (GDP per capita and unemployment rate). It is then reflected in monetary indicators (share of M0 over M1), labour market indicators (labour force participation rate and growth rate of total labour force) and the official economy (GDP growth).

Although most widely used nowadays, the MIMIC method is not without weaknesses. A comprehensive overview of them and of the deficiencies of some of its applications is provided by Breusch (2005). It has two main drawbacks. First, it treats the underground economy as a latent variable (similar to intelligence, in some psychometric applications). As Breusch (2005, p. 27) notes "the underground economy is not a latent or hypothetical quantity like intelligence; it is all too real, just difficult to measure". Second, the MIMIC method provides an index for the

latent variable (the unrecorded economy) which does not have a clear real world value. The transformation of this index into concrete real-world values is done through "benchmarking", which means that the author assumes a certain value for the unrecorded economy for a certain period, and then uses the dynamics of the estimated index to produce a series for the unrecorded economy for the whole period. This means that the MIMIC approach cannot estimate the size of the unrecorded economy, but only its dynamics.

For these two reasons, we propose to estimate the unrecorded economy through the Kalman filter. The essence of our approach is the same as the MIMIC approach – unrecorded economy is modelled as a function of certain variables, and is reflected in some other variables. The major difference is that the unrecorded economy is not a latent variable, but just unobserved. Consequently, it can be estimated by the Kalman filter algorithm. The end result is a time series of the estimated size of the unrecorded economy which is in real world terms and does not have to be "benchmarked".

To our knowledge, two studies have so far applied the Kalman filter technique for a similar purpose. Karanfil and Ozcaya (2007) estimate the "unrecorded" economy in Turkey, using a very simple model in which CO_2 emissions are a function of the forest area, the country population and the total real economic activity. Total economic activity, which is the unobserved variable in their model, is modelled as an autoregressive process of order one. After total economic activity is estimated through the Kalman filter, the "unrecorded" economy is calculated as the difference between the estimated total economic activity and the official GDP.

Arango et al. (2006) estimate the "underground" economy in Colombia, using a Kalman filter estimation in which the observation equations of the model relate the currency demand, the energy consumption and the number of self-employed people to the underground economy and a set of other variables (taxes, interest rates, inflation, official GDP etc.), and the state equation models the underground economy as a function of its lag, taxes, tariffs (to capture trafficking), minimum wages, public employees (proxy for regulations), the unemployment rate and the illicit drug manufacturing areas.

IV. Our model

Our model is similar to the models from the MIMIC literature and to the model of Arango et al. (2006). It defines the unrecorded economy as a function of the taxes, the regulatory quality, the control of corruption and the government effectiveness. Unrecorded economy, then, is reflected in the demand for currency, in the foreign currency in circulation and in the energy consumption.

The roots of the model lie in the literature on informal economy, surveyed in section II. The only real novelty of the model with respect to the existing literature is the inclusion of the foreign currency exchanged in the exchange offices which is a proxy for the foreign currency in circulation. This is an important contribution, because informal economic activities in Eastern European countries are often paid for in foreign currency, due to the high currency substitution.

More precisely, the *observation* equations of our model are as follows:

$$energy_t = \alpha_{11} + \alpha_{12} * GDP_t + \alpha_{12} * unrecorded_t + \varepsilon_t \qquad (1)$$

$$M0_t/M1_t = \alpha_{21} + \alpha_{22} * unrecorded_t + \alpha_{23} * refugees + \alpha_{24} * conversion + \alpha_{25} * trend + \mu_t \qquad (2)$$

$$cash\_exchange_t/M1_t = \alpha_{31} + \alpha_{32} * unrecorded_t + \kappa_t \qquad (3)$$

Where $energy_t$ stands for the energy consumption, GDP_t for official real GDP, $unrecorded_t$ for the unrecorded economy, $M0_t$ is the monetary aggregate M0 (i.e. reserve money), $M1_t$ is the monetary aggregate M1, $refugees$ is a dummy taking a value of one for the second, third and fourth quarter of 1999, when the Kosovo refugees crisis happened (which resulted in higher M1), $conversion$ is a dummy variable taking unitary value for 2001Q4-2002Q4, which is included to capture the conversion to euro (which increased the M0), $trend$ is a deterministic trend included to capture the gradual decline in the use of cash, due to technological factors, $cash\_exchange_t$ is the amount of foreign currency exchanged in the cash exchange offices, and $\alpha's$ are coefficients to be estimated. ε_t, μ_t and κ_t are errors in the equations, which are assumed to be white noise processes, uncorrelated among them. More precisely:

$$\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon)$$

$$\mu_t \sim i.i.d.(0, \sigma_\mu)$$

$$\kappa_t \sim i.i.d.(0, \sigma_\kappa)$$

The state equation is as follows:

$$unrecorded_t = \alpha_{41} + \alpha_{42} * unrecorded_{t-1} + \alpha_{43} * PI\_tax_t + \alpha_{44} * profit\_tax_t + \alpha_{45} *$$
$$contributions_t + \alpha_{46} * reg\_quality_t + \alpha_{47} * control\_corr_t + \alpha_{48} * gov\_effectiveness_t + \iota_t$$

(4)

Where $unrecorded_t$ is the unrecorded economy, which is unobserved, PI_tax_t is the lowest personal income tax rate, $profit\_tax_t$ is the profit tax rate, $contributions_t$ is the total rate of the social contributions, $reg\_quality_t$ is a regulatory quality index, $control\_corr_t$ is an index for the control of corruption, $gov\_effectiveness_t$ is an index for the government effectiveness, $\alpha's$ are coefficients to be estimated and ι_t is the error term, assumed to be white noise and uncorrelated with the other error terms ($\iota_t \sim i.i.d.(0, \sigma_\iota)$).

V.   Data

Quarterly data, for the period 1998-2013 are used. The exact definitions of the variables and the data sources are presented in Table 1:

Table 1: Variables' definitions and data sources

Variable	Description	Source
energy	Total energy consumption (needs) in Macedonia, in tons of oil equivalent. Annual data are interpolated at quarterly frequency using the nominal imports of energy. The correlation between these two is around 80%. Seasonally adjusted using the Census X-13 method.	Energy needs - State Statistical Office of the Republic of Macedonia (SSORM), the Statistical Yearbooks. Imports of energy – National Bank of the Republic of Macedonia (NBRM).
GDP	Real GDP, in billions of denars, constant 2005 prices. Seasonally adjusted using the Census X-13 method.	SSORM
M0	Monetary aggregate M0, in billions of current denars (i.e. nominal values).	NBRM
M1	Monetary aggregate M1, in billions of current denars (i.e. nominal values).	NBRM
cash_exchange	Foreign currency cash exchanged in the exchange offices. In billions of current denars	NBRM

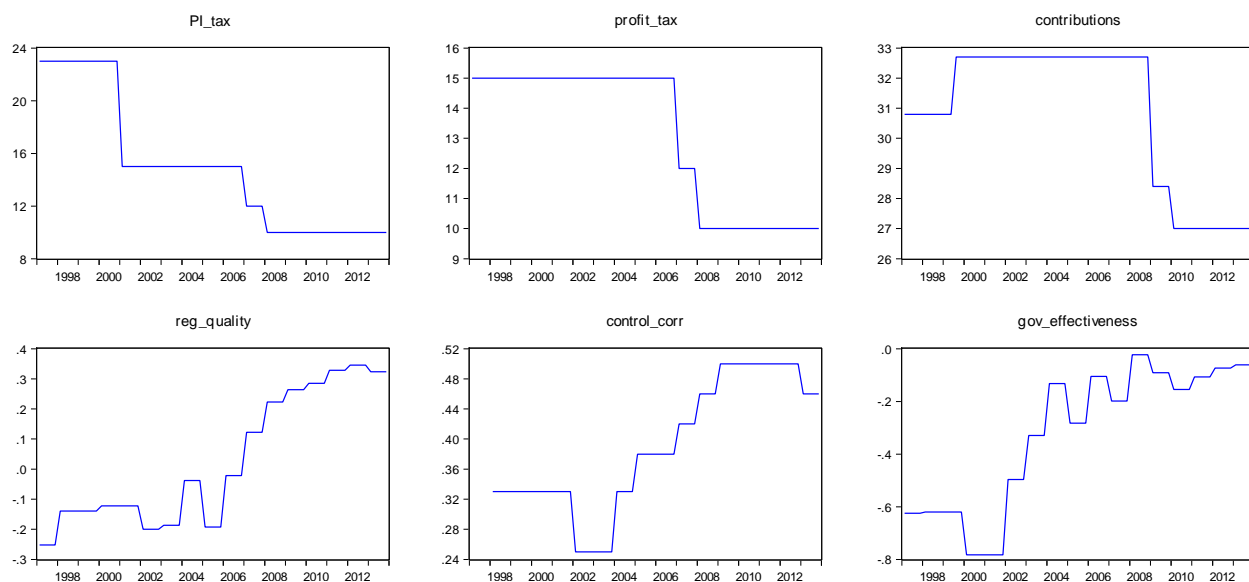|	(i.e. nominal values). Seasonally adjusted using the Census X-13 method.	
PI_tax	Personal income tax rate for the lowest income group. In percent.	Public Revenue Office of the Republic of Macedonia (PRORM)
profit_tax	Corporate profit tax rate. In percent.	PRORM
contributions	Sum of the three social contribution rates (pension, health, unemployment). In percent.	PRORM
reg_quality	Regulatory quality index. Higher value = better quality. Annual values, same for each quarter in the year.	Worldwide Governance Indicators of the World Bank (WGI)
control_corr	Control of corruption index. Higher value = better quality. Annual values, same for each quarter in the year.	Freedom House
gov_effectiveness	Government effectiveness index. Higher value = better quality. Annual values, same for each quarter in the year.	WGI

Source: Author

The tax variables that we use are the nominal tax rates, not the effective rates (tax receipts, divided by the tax basis). One may argue that the effective rates are more relevant for the behaviour of the economic agents. While this is certainly true, we have two reasons to prefer the nominal rates. The first one refers to data availability. We were able to construct effective rates for all tax variables only for the period 2002-2010. This means that, if we use effective rates, we will work with approximately 40% fewer observations. In addition, we will lose the last 3 years, which are the most important in the analysis. The second reason against the effective rates is that they already contain the unrecorded economy. The effective tax rates are a function of the tax evasion - lower evasion implies higher tax receipts, hence higher effective rates. The evasion itself is a function of the unrecorded economy. Therefore, there will be an endogeneity problem (i.e. reverse causality) in equation (4) if we use the effective rates, which would bias the results.

The variables are presented on Figures 1 and 2. Figure 1 first shows the determinants of the unrecorded economy from equation (4), i.e. the personal income tax rate (PI_tax), the profit tax rate (profit_tax), the social contributions rate (contributions), the regulatory quality index

(reg_qual), the control of corruption index (control_corr) and the government effectiveness index (gov_efectiveness).
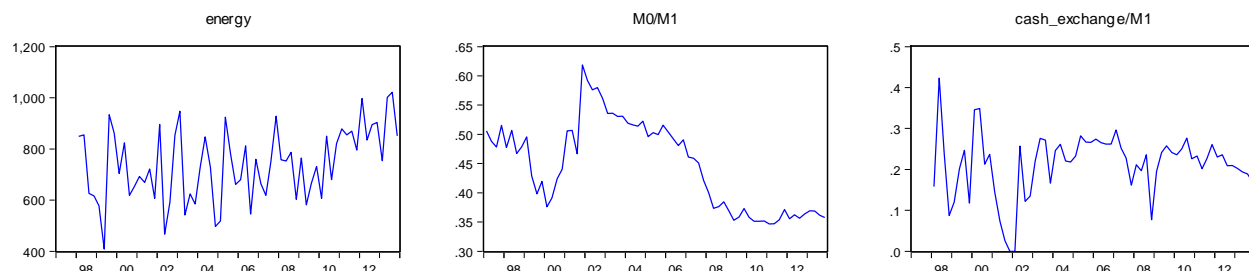
Figure 1 – Determinants of the unrecorded economy

What can be noticed is that the tax variables (PI_tax, profit_tax, contributions) have a sharp decline after 2006. The government that came to power at the end of 2006 introduced a "flat" tax system beginning in 2007, which meant establishing a single tax rate for the income and profit taxes (i.e. cancelling the progressive rates that were in power previously) and lowering the taxes (to 12% in 2007, and to 10% in 2008). The institutions variables (reg_quality, control_corr, gov_effectiveness) have increased after 2002-2003, implying that perceived regulatory quality, perceived control of corruption and perceived government effectiveness have improved. Hence, it may be expected to see a decline in the unrecorded economy around this period, both because of the lower taxes and the better institutions.

Figure 2 shows the variables in which unrecorded economy is reflected, from equations (1)-(3), i.e. energy consumption, the share of M0 in M1 and the foreign currency cash exchanged in the exchange offices. The second and third variable, the share of M0 in M1, and the share of foreign cash exchange in M1, have a downward trend, implying that the unrecorded economy is declining. The first variable, energy consumption, on the contrary, has an upward trend. This does not suggest that the unrecorded economy is increasing, though, because the official GDP

is also increasing. Indeed, the energy consumption/GDP ratio is declining, suggesting that the unrecorded economy is declining as well.

Figure 2 – Indicators of unrecorded economy

Source: Author's calculations, using data from the SSORM and the NBRM.

Descriptive statistics of the variables are presented in Table 2, while the correlation matrix is presented in Table 3. It can be noticed from Table 3 that some of the variables are very highly correlated (especially the determinants of the unrecorded economy). This may imply that it could be difficult to disentangle the individual effects of the variables, because in presence of multicollinearity, population values of the coefficients may not be estimated precisely and estimated standard errors may be high (see Gujarati, 2004, p. 374).

Table 2 – Descriptive statistics

	PI_tax	profit_tax	contributions	reg_quality	control_corr	gov_effect.	energy	M0/M1	cash_exchange	GDP
Mean	14.44	12.94	30.83	0.05	0.39	-0.30	740.94	0.44	0.21	76.37
Median	15.00	15.00	32.70	-0.03	0.38	-0.18	750.61	0.45	0.23	75.28
Max.	23.00	15.00	32.70	0.35	0.50	-0.02	1021.90	0.62	0.42	93.50
Min.	10.00	10.00	27.00	-0.20	0.25	-0.78	408.21	0.35	0.00	58.62
St.Dev.	4.69	2.40	2.49	0.21	0.09	0.26	140.73	0.08	0.08	11.18
Obs.	64	64	64	64	64	64	64	64	64	64

Source: Author's calculations.

Table 3 – Correlation matrix

	PI_tax	profit	contributions	reg_quality	control_corr	gov_effect.	energy	M0/M1	cash_exchange	GDP
PI_tax	1.00	0.79	0.52	-0.77	-0.70	-0.82	-0.25	0.42	0.03	-0.84
profit	0.79	1.00	0.78	-0.97	-0.91	-0.70	-0.35	0.83	-0.06	-0.92
contributions	0.52	0.78	1.00	-0.81	-0.76	-0.46	-0.37	0.74	-0.09	-0.71
reg_quality	-0.77	-0.97	-0.81	1.00	0.93	0.73	0.38	-0.84	0.10	0.94
control_corr	-0.70	-0.91	-0.76	0.93	1.00	0.68	0.32	-0.86	0.18	0.91
gov_effect.	-0.82	-0.70	-0.46	0.73	0.68	1.00	0.25	-0.44	0.25	0.84
energy	-0.25	-0.35	-0.37	0.38	0.32	0.25	1.00	-0.36	0.04	0.38
M0/M1	0.42	0.83	0.74	-0.84	-0.86	-0.44	-0.36	1.00	-0.23	-0.76
cash_exchange	0.03	-0.06	-0.09	0.10	0.18	0.25	0.04	-0.23	1.00	0.20
GDP	-0.84	-0.92	-0.71	0.94	0.91	0.84	0.38	-0.76	0.20	1.00

Source: Author's calculations.

VI.   Kalman filter

The Kalman filter has been proposed by Kalman (1960). It is a recursive estimator that minimizes the estimated error covariance. It estimates the unobserved variables and the model parameters through a series of predictions and corrections. It starts with some initial conditions (i.e. starting values for the model parameters, and initial mean and variance of the unobserved variable), and then predicts the unobserved and observed variables. Then, it estimates the prediction error of the observed variables and includes this in the next iteration. Values for the model parameters and the unobserved variables that maximize the likelihood function are the final estimates. It can estimate past, present and future values of the unobserved variables, even when the precise nature of the modelled system is unknown. More details about the Kalman filter can be found in Welch and Bishop (2001) for instance. The application of the Kalman filter, then, requires setting the starting values for the model parameters and setting initial values for the mean and variance of the unobserved variable. The starting values are set according to the Ordinary Least Squares (OLS) estimates of equations (1) - (4), where a proxy unrecorded economy is used, constructed as a series equal to 40% of the official GDP[2]. For the

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[2] The proxy unrecorded economy was set to be 40% of GDP, following the existing studies on unrecorded economy in Macedonia (see section VII.c). Results remain unchanged even if it is set to 30% or 50%.

parameters for which the OLS estimation gave wrongly signed coefficients, we used informed guesses[3]. The starting values that were used are presented in Table 4.

Table 4 – Starting values

Parameter	Value used	How it was set
α_{11}	370	OLS
α_{12}	5	OLS
σ_ε	150	OLS
α_{21}	1	OLS
α_{22}	-0.03	OLS
α_{23}	-0.075	OLS
α_{24}	0.06	OLS
α_{25}	0.005	OLS
σ_μ	0.05	OLS
α_{31}	0.2	OLS
α_{32}	0.002	OLS
σ_κ	27	OLS
α_{41}	8	OLS
α_{42}	0.8	OLS
α_{43}	0.05	OLS
α_{44}	0.05	Informed guess
α_{45}	0.05	Informed guess
α_{46}	-1	Informed guess
α_{47}	-1	Informed guess
α_{48}	-1	Informed guess
σ_ι	0.7	OLS

Source: Author

The initial mean of the unobserved variable (unrecorded economy) was set to 20, which is approximately one-third of the official GDP in the initial period, which is similar to the findings in Schneider et al. (2010) and Garvanlieva et al. (2012); see Section VII.c. The variance of the unrecorded economy was set to 7, which is approximately one-third of the mean.

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[3] For example, α_{44} and α_{45} are set equal to the estimated value of α_{43}

VII.   Results

a. *Unrecorded economic activity*

The estimated parameters are presented in Table 5.

Table 5 – Estimated parameters

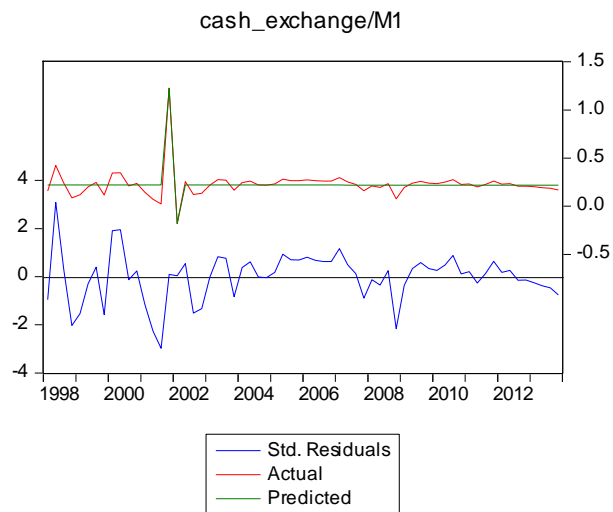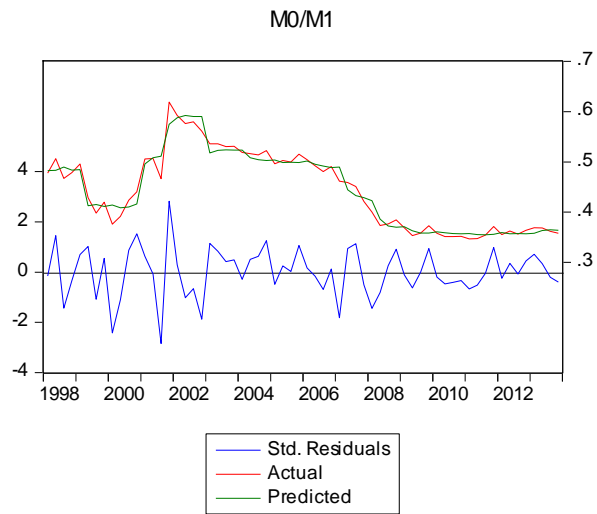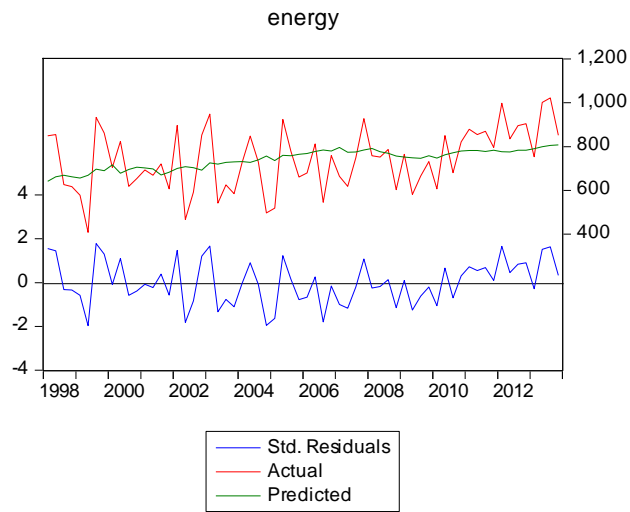| Parameter	Coefficient	p-value
α_{11}	93	0.82
α_{12}	6.98	0.07
σ_ε	17603	0.00
α_{21}	0.26	0.35
α_{22}	0.01	0.02
α_{23}	-0.075	0.00
α_{24}	0.07	0.00
α_{25}	0.0001	0.91
σ_μ	0.0001	0.00
α_{31}	0.22	0.00
α_{32}	0.0002	0.94
σ_κ	0.004	0.00
α_{41}	-1.6	0.91
α_{42}	0.4	0.13
α_{43}	-0.14	0.24
α_{44}	1.19	0.06
α_{45}	0.03	0.87
α_{46}	-1.44	0.80
α_{47}	-9	0.29
α_{48}	-1.13	0.48
σ_ι	0.7	0.00

Source: Author's estimations

We first briefly interpret the observation equations. The coefficient in front of the unrecorded economy in the energy equation, α_{12}, is significant at 10%, and indicates that if unrecorded economy increases by 1 billion of denars (in 2005 prices), energy consumption will increase by 7 tons of oil equivalent. The coefficient on the unrecorded economy in the currency demand equation, α_{22}, is significant at 5%, and indicates that if unrecorded economy increases by 1 billion of denars, the share of M0 in M1 will rise by 0.01 (1 percentage points). The coefficient on the unrecorded economy in the equation on foreign currency in circulation, α_{32}, is insignificant and rather low (0.0002), implying that the unrecorded economy affects the foreign

currency in use only marginally. This indicates that the unrecorded economy is not carried out in foreign currency, but in denars. The insignificance may also be explained by the high euroization of the economy (i.e. people store value in euros, and exchange this money in denars when paying for official economic activity).

We next interpret the state equation. Most of the coefficients are insignificant here, with exception of the profit tax coefficient, α_{44}, which can be explained by the high correlation between the explanatory variables. The coefficient on the lag of the unrecorded economy, α_{41}, is 0.4, implying moderate persistence. The coefficients on the personal income tax, α_{43}, is with an opposite sign of the expected (-0.1), suggesting that lower income tax raises the unrecorded economy. Nevertheless, the effect is rather small. The coefficient on the profit tax, α_{44}, of 1.2 suggests that if the profit tax falls by 5 percentage points, the unrecorded economy would fall by 6 billion of denars, which is a rather sizeable effect. The coefficient on the social contributions rate, α_{45}, of 0.03 suggests that lower contributions reduce the unrecorded economy, but only marginally – if contributions decline by 5 percentage points, unrecorded economy would fall by only 0.15 billion of denars. The coefficient on the regulatory quality, α_{46}, of -1.4 suggests that improvement in the quality of regulation by 0.5 index points, which is approximately the improvement between 2006 and 2011, will translate into 0.7 billion denars lower unrecorded economy. The coefficient on the control of corruption, α_{47}, of -9 implies that improvement in the control of corruption of 0.25 index points (equal to the improvement between 2004 and 2009) will lead to 2 billion denars lower unrecorded economy, which is sizeable. The coefficient on the government effectiveness, α_{17}, of -1 suggests that improvement in the effectiveness of 0.7 index points (roughly, the improvement between 2002 and 2008) will lead to a 0.7 billion denars lower unrecorded economy. All in all, it would seem that the most important factors for the unrecorded economy in Macedonia are the profit tax and the control of corruption (the second one is insignificant).

Figure 3 plots the actual values, the fitted values and the residuals for the three observation equations (equations 1-3). It can be seen that the fitted values are close to the actual ones, and the residuals seem to be well-behaved (i.e. white noise).
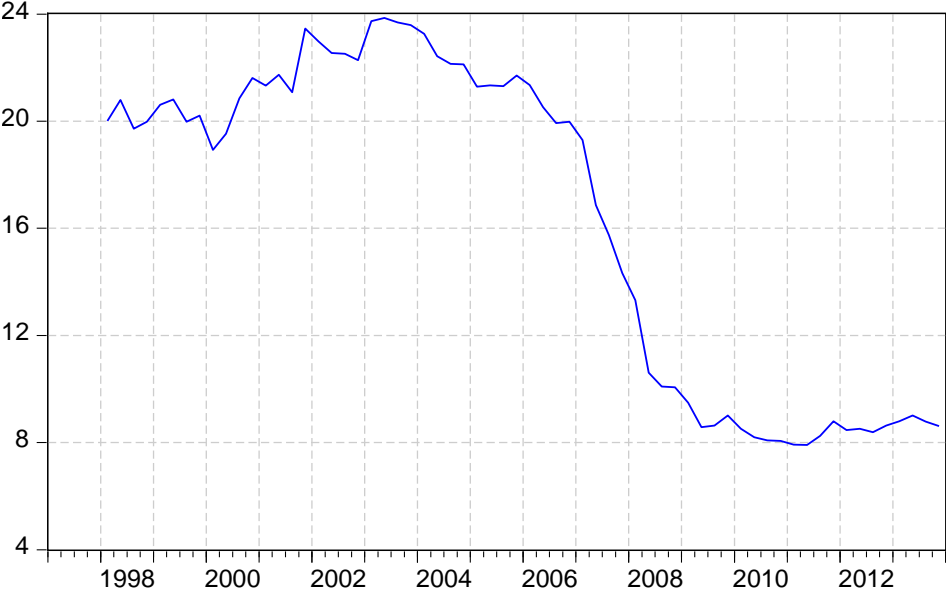
Figure 3 – Actual values, fitted values and residuals for the three observation equations

energy

M0/M1

cash_exchange/M1

Source: Author's estimation

The estimated series for the unrecorded economy are presented on Figure 4. We present the smoothed values, not the filtered (although they are very similar). The series are in the same units as the GDP (i.e. in billions of denars, in constant 2005 prices). The series suggests that unrecorded economy was stagnating between 1998 and 2000. Then it started rising, for two years, after which it started falling, first mildly (between 2002 and 2006) and then rapidly (2006-2008). Since 2009, it has been stagnating, again.

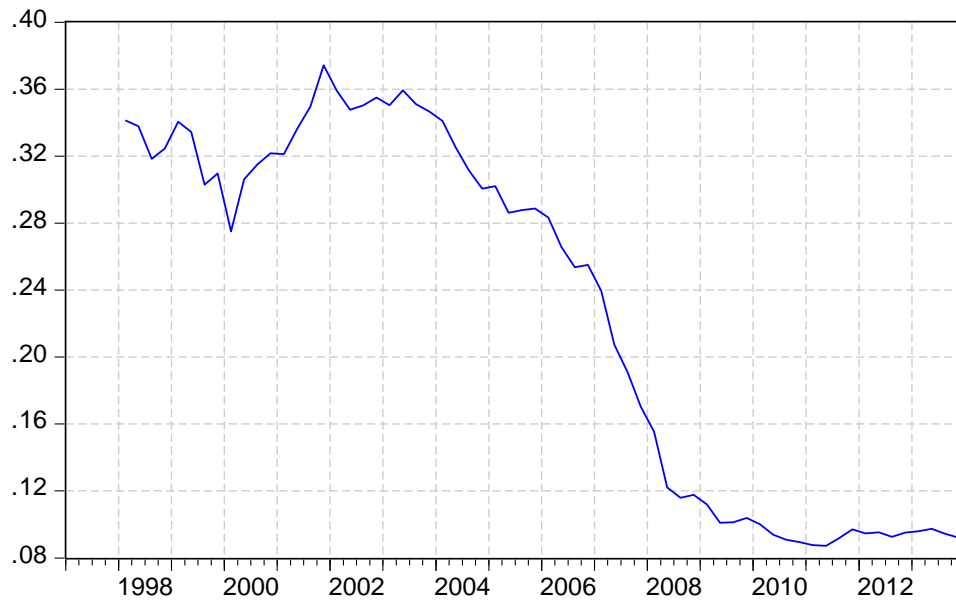Figure 4 – Estimated unrecorded economy, billions of denars, constant 2005 prices

Source: Author's estimation

Figure 5 shows the unrecorded economy (the smoothed series) as a percentage of the official GDP. The dynamics here are similar to those from the previous figure. The size of the unrecorded economy was approximately 32% of GDP between 1998 and 2003, and then started falling, reaching 15% of GDP in 2008. Since then, it has declined marginally, to 10% in 2013.

The movements in the unrecorded economy are in accordance with the dynamics of its determinants, which all push for a downward trend in the unrecorded economy after 2003 (taxes, contributions, regulatory quality, government effectiveness and control of corruption). Due to the high correlation between the explanatory variables and the insignificance of many of them, we would refrain from assessing the relative contribution of the determinants.
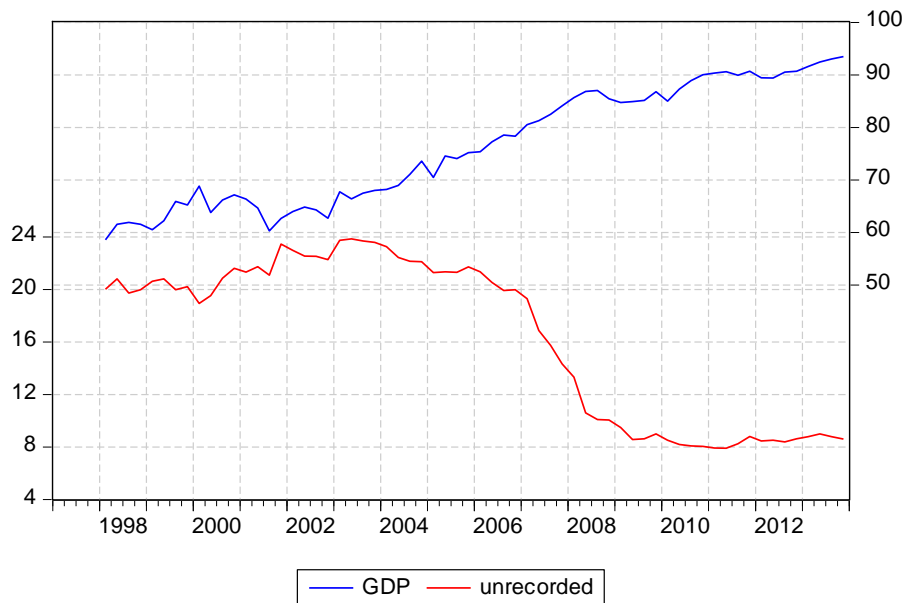
Figure 5 – Unrecorded economy as a share of official GDP

Figure 6 plots the unrecorded economy along the official GDP. It can be seen that the two series are inversely related. This may imply that there is a transfer between them – unrecorded economy falls because businesses turn from informal to formal, which then raises the official GDP.
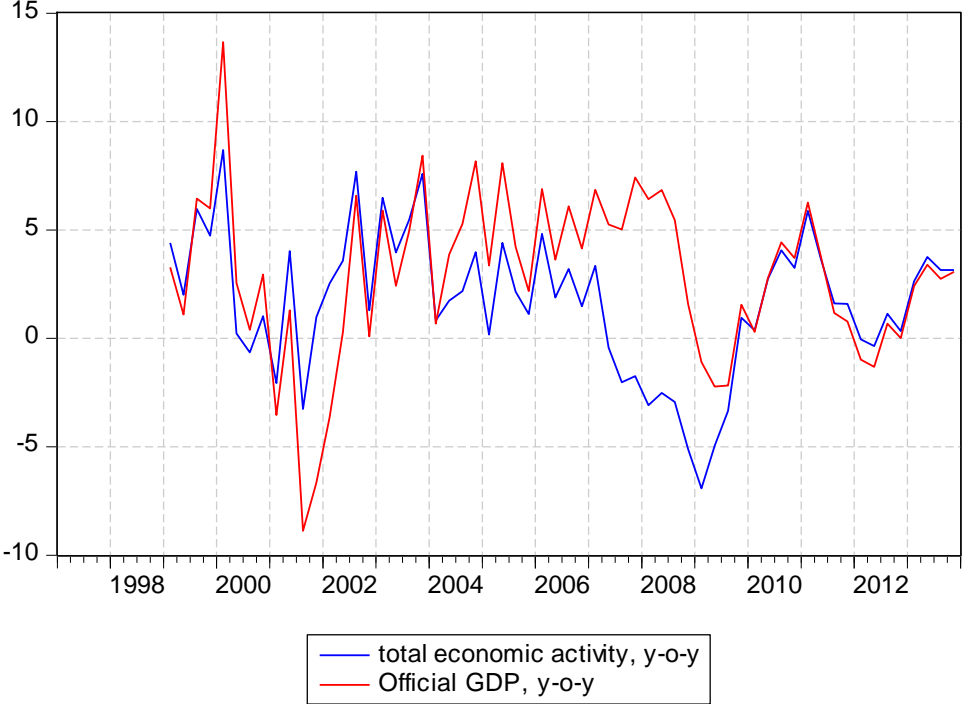
Figure 6 – Unrecorded economy and official GDP

Figure 7 compares the official GDP and the total economic activity (i.e. the official GDP, plus the unrecorded economy). For clarity, we present the year-on-year growth rates. The two series exhibit similar movements, though they differ markedly during 2004-2005 and especially during 2006-2009. During 2004-2005, the average annual growth rate of the official GDP is 4.5%, while the growth rate of the total economic activity is around 2%. Even more pronounced are the differences during 2006-2009. From the second half of 2006, until the end of 2009, the average annual growth rate of the official GDP is approximately 3.9%. During the same period, the average annual growth rate of the total economic activity is approximately 0%, considering the slowdown in the unrecorded activity.

Figure 7 – Official GDP and total economic activity, year-on-year growth rates

Source: Author's estimation

b. *Robustness checks*

In order to assess the stability of the estimates, we eliminate the explanatory variables from the state equation one by one. The estimated coefficients are presented in Table 6. One can see that there are virtually no changes in the coefficients of the three observation equations. Turning to the state equation, the coefficients here look rather stable, too. The coefficient on
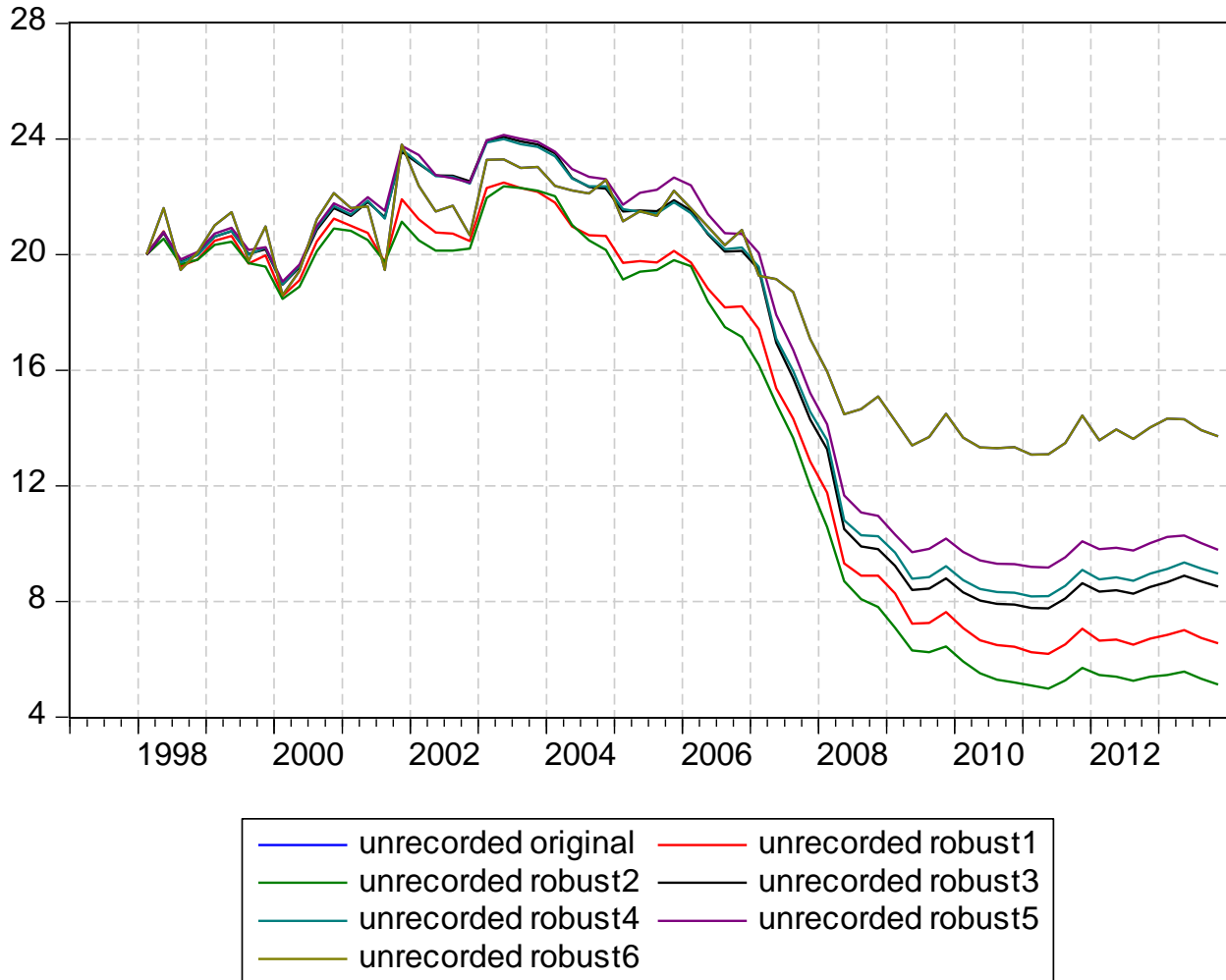
the lag of the unrecorded economy, α_{42}, ranges between 0.2 and 0.7, which is close to the original value of 0.4. The coefficient on the income tax, α_{43}, whose original value was unexpectedly negative (-0.14) becomes positive once the profit tax is excluded, suggesting that the negative sign in the original specification is due to the high correlation between the profit and income taxes. The coefficient on the profit tax, α_{44}, is very stable, ranging between 0.9 and 1.2. The coefficient on the social contributions, α_{45}, whose original value was 0.03, is always around zero. The coefficient on the regulatory quality, α_{46}, ranges between -1.3 and -3.4 in five of the specifications. It becomes -6.8 only when the profit tax is excluded, which is due to the very high correlation between these two variables. The coefficient on the control of corruption is very robust, α_{47},ranging between -5.8 and -9, as well as the coefficient on the government effectiveness, α_{48}, which ranges between -0.5 and -1.2. Therefore, we can say that these robustness checks yield support to the findings above, that the most important determinants of the unrecorded economy are the profit tax and the control of corruption.

The resulting estimates of the unrecorded economy are shown on Figure 8. It can be noted that all the series are qualitatively very similar – they show a modest increase between 1998 and 2002, then start declining mildly until 2006, then start falling rapidly until 2009, and stagnate afterwards. Their magnitudes are also reasonably similar. For instance, excluding the most extreme series (produced by the sixth robustness exercise), the estimates for the last quarter of 2013 range between 4 and 10 billion of denars, which is approximately 4% and 10% of the official GDP.

Table 6 – Robustness checks

	Original	Robustness 1	Robustness 2	Robustness 3	Robustness 4	Robustness 5	Robustness 6
α_{11}	93	39	13	104	79	131	171
α_{12}	6.98	7.66	8.01	6.86	7.12	6.52	6.01
σ_{ε}	17603	17589	17729	17657	17600	17527	17114
α_{21}	0.26	0.23	0.24	0.26	0.26	0.26	0.11
α_{22}	0.01	0.01	0.01	0.01	0.01	0.01	0.02
α_{23}	-0.075	-0.075	-0.075	-0.075	-0.075	-0.075	-0.075
α_{24}	0.07	0.08	0.08	0.07	0.07	0.07	0.07
α_{25}	0.0001	0.0008	0.0009	0.0001	0.0000	-0.0001	-0.0001
σ_{μ}	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
α_{31}	0.22	0.22	0.22	0.22	0.22	0.21	0.21
α_{32}	0.0002	0.0001	0.0001	0.0002	0.0002	0.0003	0.0006
σ_{κ}	0.004	0.004	0.004	0.004	0.004	0.004	0.004
α_{41}	-1.6	-1.5	8.5	-1.1	-2.5	-2.4	4.4
α_{42}	0.4	0.4	0.7	0.4	0.4	0.5	0.2
α_{43}	-0.14		0.01	-0.15	-0.15	-0.16	-0.09
α_{44}	1.19	0.87		1.19	1.28	1.05	0.95
α_{45}	0.03	0.07	-0.09		0.04	-0.03	0.08
α_{46}	-1.44	-2.55	-6.79	-1.48		-3.44	-1.33
α_{47}	-9.0	-9.0	-5.8	-8.9	-9.8		-7.1
α_{48}	-1.13	-0.65	-0.46	-1.11	-1.22	-1.17	
σ_{ι}	0.7	0.7	0.7	0.7	0.7	0.7	0.7

Figure 8 – Alternative estimates of the unrecorded economy

c. Comparison with other studies for Macedonia

Several existing studies have estimated the unrecorded economy in Macedonia. Schneider et al. (2010) estimate it using the MIMIC approach. They find that the unrecorded economy has declined from 39% of GDP in 1999, to 35% in 2007. Garvanlieva et al. (2012) apply the electricity consumption method and the MIMIC method. According to the electricity consumption method, the unrecorded economy in Macedonia is estimated to have declined from 34% of GDP in 2000, to 24% in 2010. According to the MIMIC method, the unrecorded economy increased from 34% of GDP in 2003, to 52% in 2007, and decline to 47% in 2011. Elgin and Oztunali (2012) apply a dynamic general equilibrium model, finding that the

unrecorded economy in Macedonia has fluctuated between 34%-35% of GDP during 2000-2008.

Our results are similar to these studies in two respects. First, the size of the unrecorded economy in our initial period (1998Q1), of approximately 34% of GDP, is similar to the estimates from these studies. Second, both our results and the results from the existing studies indicate a downward trend in the unrecorded economy.

Our results, however, differ significantly from the existing studies regarding the current estimates of the unrecorded economy. Our estimate of 12% of GDP is much lower than the lowest estimate from the existing studies, of 24% in 2010 (Garvanlieva et al, 2010).

d. Unrecorded employment and true unemployment

Having estimated the unrecorded economy, we can now proceed to estimating the unrecorded employment. To do this, we multiply the unrecorded economic activity (in billions of denars) with the number of employees per unit of formal activity from the official data (employees/GDP). We use a wide range of values for the employees/GDP ratio, obtained from the sectors which are believed to hold most of the unrecorded economy. In particular, following Dzekova et al. (2014), we posit that the unrecorded economy in Macedonia is most prevalent in the following five sectors: trade, hotels and restaurants, transportation, agriculture and construction. We use three different values for the number of employees per unit of GDP. The first is from trade, and hotels and restaurants, the second is from trade, hotels and restaurants, transportation and construction, and the third is from all the five sectors together. The average values refer to the period 1998-2013. The specific values are presented in Table 7. It can be seen that the lowest of them is approximately equal to the value for the total economy (given in the last row), while the highest one is approximately one quarter higher.
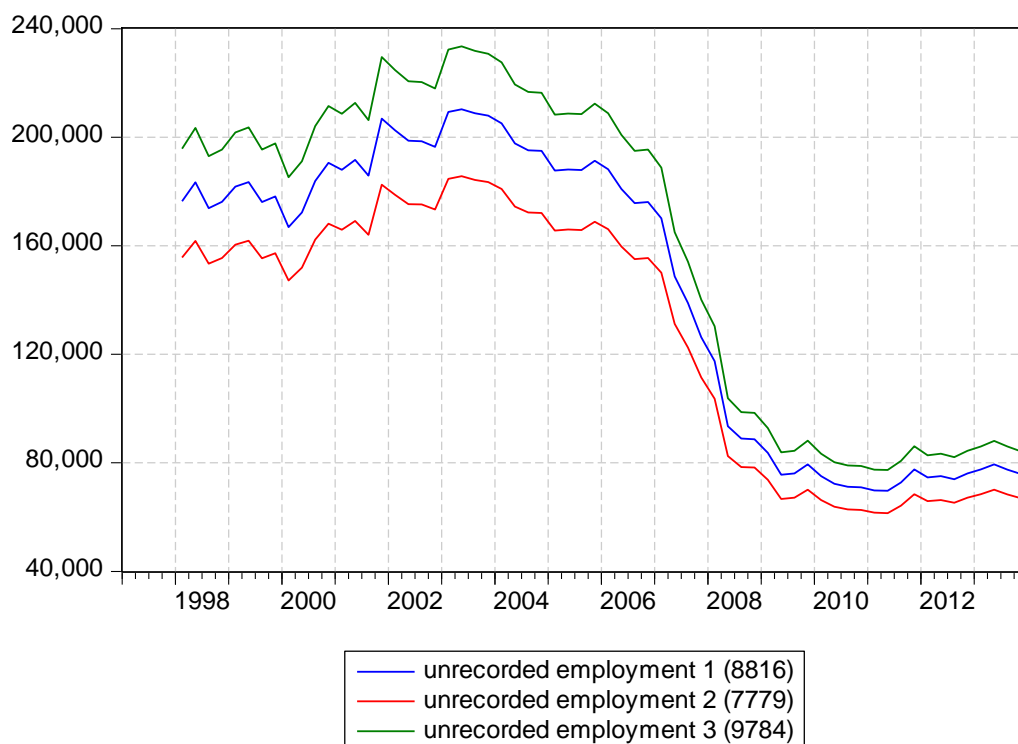
Table 7 – Employees per 1 billion denars of value added (in 2005 constant denars)

Sector	Employees per unit of output
Trade and hotels and restaurants	8816
Trade, hotels and restaurants, transportation, construction	7779
Trade, hotels and restaurants, transportation, construction, agriculture	9784
Total economy	7267

Source: Author

The series for the unrecorded employment are depicted on Figure 9. It can be seen that since 2009, the unrecorded employment has been in the range 70,000-90,000. This is much lower than the value from 1998-2006, when unrecorded employment was ranging between 160,000 and 230,000.

Figure 9 – Unrecorded employment

Source: Author's estimation

The total employment, equal to the sum of the unrecorded employment and the employment that the State Statistical Office publishes, is shown on Figure 10. We show this series only after 2004, because prior to 2004 there are no quarterly data on official employment, and because there were structural changes in the official employment series after 2004. The total employment has declined in 2004, to rise immediately in 2005. Between 2006 and 2010 it has stagnated mainly. After 2010 it started rising mildly, which has been the trend until the end of the sample (2013).
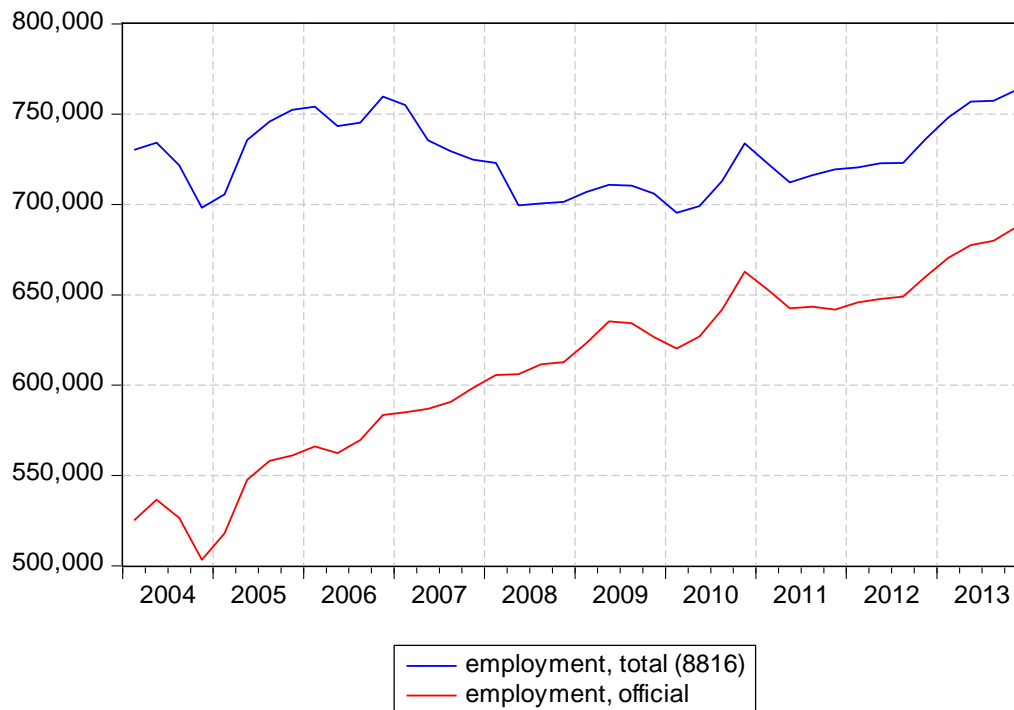
Figure 10 – Total employment

Source: Author's estimation

Figure 11 compares the total employment (the first version) with the official one. Obviously, the total employment is higher than the official one. Apart from this, the movements of the two series are rather similar, the only major difference being the dynamics between 2006 and 2010, when the official employment was growing, but the total one was stagnating. This is explained by the decline in the unrecorded employment during this period. In other words, the increase in the official employment between 2006 and 2010 was partially due to the transfer from informal to formal employment.
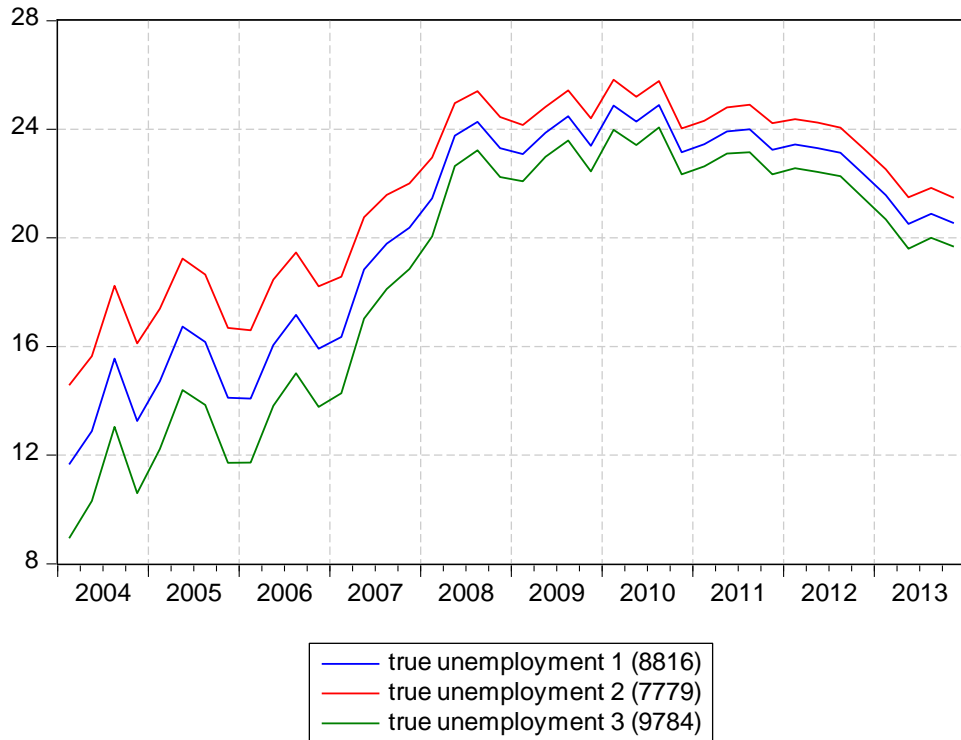
Figure 11 – Official and total employment

Source: Author's estimation

Finally, using the series for the total employment, we can calculate the true rate of unemployment in Macedonia, using the following formula:

$$true\ unemployment\ rate = \frac{(active\ population - total\ employment)}{(active\ population)} * 100 \qquad (5)$$

The three series for the true unemployment are shown on Figure 12. The true unemployment was rising between 2004 and 2008, from 10-15% in 2004, to 23-25% in 2008. It was stagnating from the second half of 2008 until the second half of 2010, and it started falling from the end of 2010. Currently, it is estimated to be somewhere between 20% and 22%.
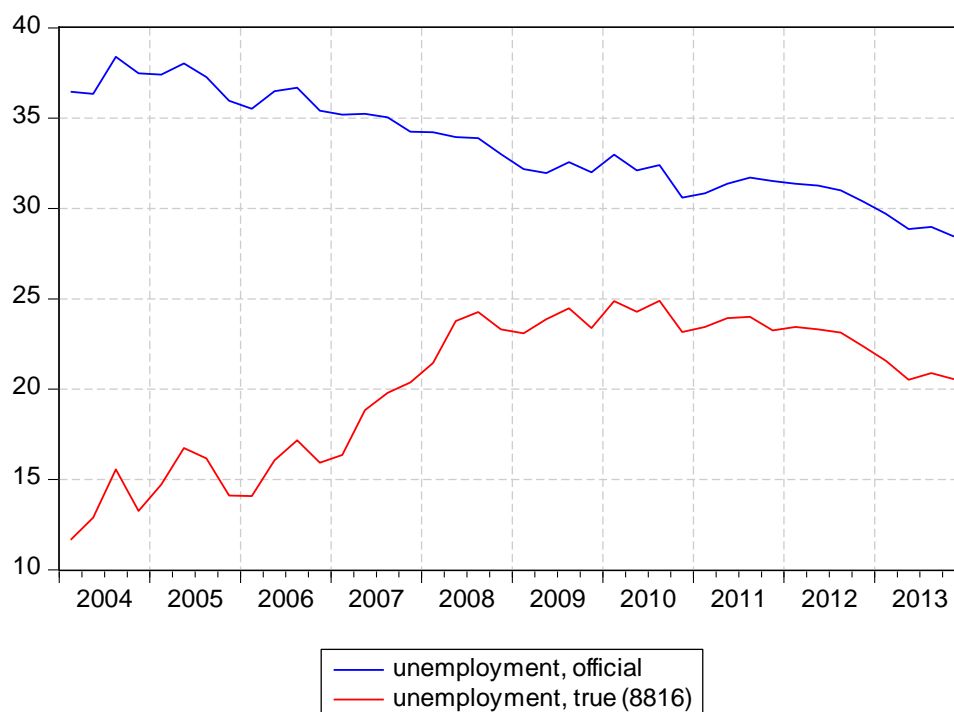
Figure 12 – True rate of unemployment

Source: Author's estimation

Figure 13 plots the official rate of unemployment with the true rate of unemployment. The official rate is much higher than the true one, but the gap is declining. The difference between the two was more than 20 percentage points in 2004, while in 2013, it was around 8 percentage points. Apart from the level, the two series have different trends, especially in the first part of the period. The official rate of unemployment is declining between 2004 and 2008, (from 36% to 33%), but the true rate of unemployment is rising (from 12% to 24%). This is due to the decline in the unrecorded employment, i.e. due to the transfer from informal to formal employment. In the period afterwards, both rates follow gradual declining path.

Figure 13 – Official and true unemployment

Source: Author's estimation

VIII. Conclusions

The unrecorded economy is important for three main reasons. First, workers engaged in it do not enjoy the same rights as the workers working in the formal economy, such as health and pension insurance. Second, companies working in it do not pay taxes, which leaves governments without some revenues, which then translates into lower public spending and lower standard of living. Hence, authorities should know the unrecorded economy, and should take measures to fight it.

But, these two reasons are not the only reasons to study the unrecorded economy. Unrecorded economy can be of interest to central bankers, too, because it can affect the inflation, their main object of interest. Similarly, the unrecorded economy may be interesting to the general public, especially in countries like Macedonia, where the official unemployment rate has constantly been above 30% since early 1990's. Simply put, people do not believe that the

unemployment is really above 30% for two decades, and may like to know the true rate of unemployment, which includes the unrecorded employment.

This paper estimates the unrecorded economic activity and the unrecorded employment in Macedonia, during 1998-2013. It models the unrecorded economy as unobserved variable, driven by the tax "burden" (income tax and social contributions), the quality of the business regulation and the quality of the governance. Then, unrecorded economy is reflected in the cash in circulation, the foreign currency in circulation and the energy consumption. The unrecorded economy is estimated using the Kalman filter algorithm.

After the unrecorded economic activity is estimated, the unrecorded employment is calculated by multiplying the unrecorded activity with several values for the employment/GDP ratio, obtained from the sectors where the unrecorded economy is believed to be most prevalent. Once unrecorded employment is calculated in this way, it is summed with the official employment, to find the total employment in the economy. This series is then used to calculate the true rate of unemployment in the Macedonian economy.

Results point out that the unrecorded economic activity has declined between 1998 and 2013, from 34% of GDP in 1998, to 10% of GDP in 2013. The bulk of the decline happened between 2004 and 2009, due to the improved control of corruption and the lower profit tax. Consequently, the unrecorded employment declined too, from 160-200,000 in 1998, to 70-90,000 in 2013. Finally, the true rate of unemployment is currently estimated at around 20-22%, which is much lower than the official unemployment of 28%.

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